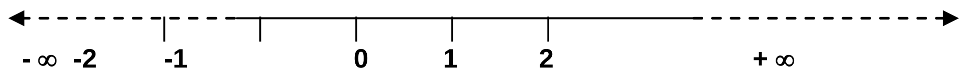
Understanding Numbers

Numbers and Number line

A measurement carried out, of any quantity, leads to a meaningful value called the number. This value may be positive or negative depending on the direction of the measurement and can be represented on the number line.



Positive and Negative Numbers

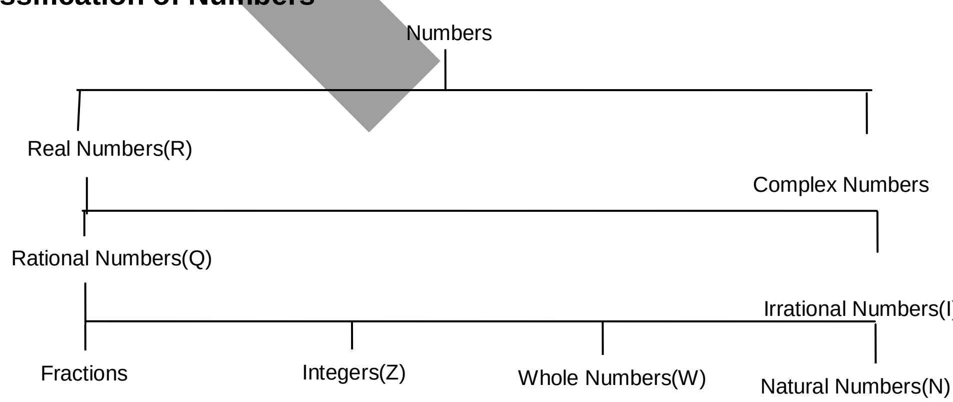
If one considers the filling of a water tank upto a level as a typical example the concept of direction of measurement can be better understood. If the tank is filled upto 2 m, the measurement of filling will be 2 m. Further if water is used up and the tank is emptied, the measurement of filling will be -2 m . If one is measuring the emptying of a tank, if the tank is filled upto 2 m, the measurement of emptying will be - 2 m. Further if water is used up and the tank is emptied, the measurement of emptying will be 2 m .

Therefore one must realize that positive measurement and negative measurement are in opposition of direction with respect to one another and simply represent the opposition in the nature of the physical variables measure

Rules to remember

1. Multiplication of two positive numbers gives a positive number.
2. Multiplication of two negative numbers gives a positive number.
3. Division of two positive numbers gives a positive number.
4. Division of two negative numbers gives a positive number.
5. Addition of two positive numbers gives a positive number.
6. Addition of two negative numbers gives a negative number.
7. Subtraction of two positive numbers gives a positive or a negative number.
8. Subtraction of two negative numbers gives a positive or a negative number.
9. Multiplication of a positive number and a negative number gives a negative number.
10. Division of a positive number and a negative number gives a negative number.
11. Addition of a positive number and a negative number gives a positive or a negative number.
12. Subtraction of a positive number and a negative number gives a positive or a negative number.

Classification of Numbers



Letters in brackets denote the standard symbols for the respective numbers.

Real Numbers

Those numbers which are seen to measure tangible variables in the strictly physical sense are known as real numbers. These can be further classified as rational numbers and irrational numbers.

A) Rational Numbers: Numbers which can be expressed in the form of p/q; where p and q are both integers and $q \neq 0$ are called rational numbers. Rational numbers when expressed in decimal form are either terminating or recurring. These are of four types.

1. **Fractions:** A number which can be represented as the ratio of two values x/y, where $y \neq 0$, is called a fraction.
   e.g. 2/3, 5/8, -3/7
2. **Integers:** All numbers that are not fractions are integers. That is from $-\infty$ to $+\infty$.
   e.g. -5, 0, 2
3. **Whole numbers:** All numbers that are not fractions or negative numbers are whole numbers. That is from 0 to $+\infty$.
   e.g. 0, 2, 7
4. **Natural numbers:** All numbers that are not fractions, negative numbers, or zero are natural numbers. That is from 1 to $+\infty$.
   e.g. 1, 4, 7

B) Irrational Numbers: An irrational number is that number which gives an approximate number in the form of a fraction or a decimal. That is, the numbers whose decimal forms are non terminating and non recurring e.g. $\sqrt{2}, \sqrt[3]{3}, \ldots, \Pi$, e. The numbers Π and e are the irrational numbers which are not of the form $\sqrt[n]{m}$. These are known as the ***Transidental*** **Irrational** numbers

No number can be both rational and irrational.

Complex Numbers

The system of real numbers is inadequate as it contains no number whose square is a negative number. So complex numbers or imaginary numbers were employed to find solutions to quadratic equations. The generalised complex number n is of the form n = a + bi, where a and b are any real numbers and $i = \sqrt{-1}$ ($i^2 = -1$), is known as the imaginary unit, a is the real part of n and b is it's imaginary part. If a = 0, then the number is known as purely imaginary number. If b = 0 then the number is purely real number

These numbers do not figure in arithmetic operations.

Odd and Even Numbers

All numbers divisible by 2 are even numbers. All the numbers, which are not divisible by 2, except 0, are odd numbers. 0 is neither an even number nor an odd number.

Rules to remember

1. Addition of two odd numbers gives an even number.
2. Addition of two even numbers gives an even number.
3. Addition of an odd and an even number gives an odd number.
4. Subtraction of two odd numbers gives an even number.
5. Subtraction of two even numbers gives an even number.
6. Subtraction of an odd and an even number gives an odd number.
7. Multiplication of two odd numbers gives an odd number.
8. Multiplication of two even numbers gives an even number.
9. Multiplication of an odd and an even number gives an even number.
10. Division of an odd number (divisible) by an odd number gives an odd number.
11. Division of an even number (divisible) by an odd number gives an even number. An odd number is not divisible by an even number.

Prime, Composite and Co-Prime Numbers

Prime number

A prime number is a number greater than 1 which has no factor other than itself and unity. e.g. 2, 3, 5, 7, 11, 13, 17,........

Composite number

A composite number is a number which has other factors besides itself and unity.

e.g.

number	factors
14	1 X 14, 2 X 7
36	1 X 36, 2 X 18, 3 X 12, 4 X 9, 6 X 6

Co-prime numbers

Two numbers are co-prime to one another if they have no other factors common except unity.
e.g. 35 and 12

number	factors
35	1 X 35, 5 X 7
12	1 X 12, 2 X 6, 3 X 4

Rules to remember

1. The number 1 is neither prime nor composite.
2. 2 is the only even number which is prime.
3. A number is said to be a factor when it divides the other number exactly. (e.g. 4 & 9 are factors of 36).
4. A number is said to be a multiple when it is exactly divisible by that number. (e.g. 35 is a multiple of 5 and 7.

Consecutive Integers

Consecutive integers are numbers differing by 1 in a series of numbers in ascending order. (e.g. 13,14,15).

Consecutive **odd** integer are numbers differing by 2 in a series of numbers in the ascending order, starting with an odd integer.(e.g. 5, 7, 9).

Consecutive **even** integer are numbers differing by 2 in a series of numbers in the ascending order, starting with an even integer. (e.g. 4, 6, 8,).

Basic Arithmetic Operations

Addition

This is denoted by the symbol '**+**' and is the process of finding the number which is equal to two or more quantities taken together.

Subtraction

This is denoted by the symbol '**-**' and is the process of finding what quantity is left when a smaller number is taken from a greater one.

Multiplication

This is denoted by the symbol 'X' and is the process of finding the sum of a given number of repetitions of the same number.

Division

This is denoted by the symbol '÷' and is the process of finding how often a number is contained in another number.

VBODMAS

In resolving the value of a given expression the various operations must be performed in the given order.

1. Viniculum or Bar	V
2. Removal of brackets in the order ( ), { }, [ ].	B
3. Of	O
4. Division ÷	D
5. Multiplication X	M
6. Addition +	A
7. Subtraction -	S

Tests of Divisibility

1. A number is said to be divisible by 2 if its unit digit is 0 or even.
2. A number is said to be divisible by 3 if the sum of its digits is divisible by 3
3. A number is said to be divisible by 4 if its last two digits are 0 each or the number formed by them is completely divisible by 4.
4. A number is said to be divisible by 5 if its last digit is 0 or 5.
5. A number is said to be divisible by 6 if the number is divisible by 2 and 3.
6. A number is said to be divisible by 8 if its last three digits are 0 each or the number formed by them is divisible by 8.
7. A number is said to be divisible by 9 if the sum of its digits is divisible by 9.
8. A number is said to be divisible by 10 if its unit digit is 0.
9. A number is said to be divisible by 11 if the difference of the sum of its digits in the odd places and the sum of its digits in the even places is either 0 or a multiple of 11.
10. A number is said to be divisible by 16 if its last four digits are 0 each or the number formed by them is divisible by 16.

Rules to remember

A number n divisible by a number m can be split into two parts both divisible by m or two parts both non-divisible by m. No number n divisible by a number m can be split into two parts such that if one is divisible by m the other is non-divisible by m.

e.g.

a + b such that if a is divisible by m, b is divisible by m
p - q such that if p is divisible by m, q is divisible by m
r +s such that if r is non-divisible by m, s is non-divisible by m
x - y such that if x is non-divisible by m, y is non-divisible by m

Ex. Simplify (a + 2b - 3c) + (3a - b + 2c) - (c - 2a + 3b)

Sol. (a + 2b - 3c) + (3a - b + 2c) - (c - 2a + 3b)
= a + 2b - 3c + 3a - b + 2c - c + 2a - 3b.
= a + 3a + 2a + 2b - b - 3b - 3c + 2c - c
= 6a - 2b - 2c
= 2(3a - b - c)

Ex. Expand (x - 2y + 3z) (2x - y - 2z)

Sol. (x - 2y + 3z) (2x - y - 2z)
$= 2x^2 - xy - 2xz - 4xy + 2y^2 + 4yz + 6xz - 3zy - 6z^2$
$= 2x^2 + 2y^2 - 6z^2 - 5xy + yz + 4zx$

Ex. Find the remainder when 1234567 is divided by 11.

Sol. Procedure to find our the remainder when a number is divided by 11:

- Find the number of digits. Remember that position of digits has to be taken from Left to Right.
  - If n is EVEN, find D as
  D = Sum of digits at even places – Sum of digits at odd places
  - If n is ODD, find D as
  D = Sum of digits at odd places – Sum of digits at even places
- Now divide D by 11 and find the remainder R.
- If R > 0, R is the final Remainder.
If R < 0, 11 + R is the Remainder.

Here the number of digits is 7, which is odd. So D = (1 + 3 + 5 + 7) – (2 + 4 + 6) = 16 – 12 = 4. If we divide 4 by 11, the remainder obtained is 4 > 0. So answer is 4.

Exercise A

1. Simplify: $\frac{1}{3}\left[\frac{1}{4}+\left\{\frac{2}{3}-\left(\frac{1}{32}\div\overline{\frac{1}{8}-\frac{1}{16}}\right)\right\}\right]$

2. Simplify: $5\frac{1}{2}-\left[\frac{1}{2}\div\left\{\frac{3}{4}-\frac{1}{2}\left(\frac{2}{3}-\frac{1}{6}-\frac{1}{8}\right)\right\}\right]$

3. Simplify: $2\frac{1}{2}$ of $\frac{3}{4}\times\frac{1}{2}\div\frac{3}{2}+\frac{1}{2}\div\frac{3}{2}\left(\frac{2}{3}-\frac{1}{2}\text{ of }\frac{2}{3}\right)$

4. Simplify: $\frac{(5\,3/4)-(3/7)\text{ of }(15\,3/4)+(2\,2/35)\div(1\,\,11/25)}{(3/4)\text{ of }(7\,3/7)-(5\,3/5)\div(3\,\,4/15)}$

5. Simplify: $\frac{(2/3)\div(5/6)\text{ of }(3/4)}{(2/3)\div(3/4)\times(5/8)}\div\frac{(3\,1/5)}{(3\,1/3)}\text{ of }\frac{375}{1000}$

6. Simplify: $\frac{(1/2)\div(1/4)\text{ of }(1/6)}{(1/2)\div(1/4)\times(1/6)}+\frac{(1/2)-(1/4)\times(1/6)}{\{(1/2)-(1/4)\}\times(1/6)}-\frac{2.3}{.05}$

7. Simplify: $\frac{(4\,1/7)-(2\,1/4)}{(3\,1/2)+(1\,1/7)}\div\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{5-\cfrac{1}{5}}}}$

8. Simplify: $\cfrac{1}{2-\cfrac{2}{2+\cfrac{4}{2-\cfrac{5}{2+\cfrac{6}{7}}}}}$

9. Simplify: $\frac{(2/3)\div(3/4)\text{ of }(5/6)}{(2/3)\div(3/4)\times(5/6)}+(2/3)\times1.24-\{(3/5)-(1/3)\}$

10. Simplify: $\frac{.45\times[.73-\{(.35\text{ of }.9\div.15\text{ of }4.2-.05)-.4\}]}{1.53\times.35}\div\frac{.25}{.315}$

11. (i) If X381 is divisible by 11, find the value of smallest natural number X ?
(ii) If 381Y is divisible by 9, find the value of smallest natural number Y ?

12. What will be the remainder obtained when $(1234567890123456789)^{24}$ is divided by 6561?

13. Find the smallest natural number ‘n’ such that n! is divisible by 990?

14. A number is formed by writing the first 24 natural numbers consecutively. What will be the remainder if this number is divide by 9?

15. What is the remainder if 1237 x 1239 x 1241 x 1243 is divided by 30?

16. Choose the greatest fraction: 3/7, 5/9, 7/12

17. Choose the smallest fraction: 41/ 52, 32/ 39, 54/ 65

18. What is the remainder in division a^n / (a + 1) if (a) n is even (b) n is odd? Both a and n are natural numbers.

19. What is the remainder in division of 858585……10 times by 11?

20. What is the factorial of the least two-digit natural number having highest power of five, seven more than the highest power of seven in it?

Answers

1. 5/36	**2.** -349/18	**3.** 1 5/8	**4.** 1/9	**5.** 5 1/3
6. 1	**7.** 1	**8.** 9/17	**9**. 2	**10.** 0.72
11. (i) 7(ii) 6	**12.** 0	**13.** 11	**14.** 3	**15.** 9
16. 7 / 12	**17.** 41/ 52	**18. (a)** 1	**(b)** a	**19.** 3
20. 75!				

HCF, LCM

Highest Common Factor

The Highest Common Factor (H.C.F.) or the Greatest Common Divisor (G.C.D.) or the Greatest Common Measure (G.C.M.) of two numbers is the greatest number that divides each one of them exactly.

Finding H.C.F.

By Factor Method

Ex. Find the G.C.D. of 16 & 56
Factors of 16 are 1,2,4,8,16
Factors of 56 are 1,2,4,7,8,14,28,56
The common factors of 16 & 56 are 1,2,4,8. The G.C.D. of 16 & 56 is 8

By Division Factor Method

- **For two given numbers**
  Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Keep dividing the earlier divisor by the remainder last obtained, till a remainder 0 is obtained. The last divisor obtained in this fashion is the H.C.F. of the two given numbers.

- **For more than two given numbers**
  Choose any two numbers and find their H.C.F. The H.C.F. of these two numbers and the third number gives the H.C.F. of the three numbers and likewise.

Ex. Find the H.C.F. of 6851, 9061 & 18462
Take any two numbers say, 6851 & 9061

```
6851 )  9061 ( 1
        6851
        ----
        2210 )  6851 ( 3
                6630
                ----
                221 ) 2210 (10
                      2210
                      ----
                         0
```

Thus H.C.F. of 6851 & 9061 is 221.
Proceeding in a similar way we get the H.C.F. of 221 & 18462 as 17.
Thus the H.C.F. of all the given numbers is 17.

Least Common Multiple

The least number, which is exactly divisible by each of the given numbers, is called their Least Common Multiple. (L.C.M.)

Ex. Find the L.C.M. of 8 & 18
Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72,...
Multiples of 18 are 18, 36, 54, 72,...
Thus the L.C.M. of 8 & 18 is 72

Finding L.C.M.

By Factor Method

L.C.M. of given numbers can be found by resolving each of them into prime factors and then taking the product of highest powers of all the factors, that occur in these numbers.

Ex. Find the L.C.M. of 24, 54 & 70
$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$
$70 = 2 \times 5 \times 7$
L.C.M. $= 2^3 \times 3^3 \times 5 \times 7 = 7560$

By Division Method

2	24	54	70
2	12	27	35
2	6	27	35
3	3	27	35
3	1	9	35
3	1	3	35
5	1	1	35
7	1	1	7
	1	1	1

L.C.M. = 2 x 2 x 2 x 3 x 3 x 3 x 5 x 7 = 7560

Rules to remember

1. Product of two given numbers is equal to the product of their H.C.F. & L.C.M.

2. Given two numbers their L.C.M. is equal to Product of the two numbers divided by the H.C.F. of these numbers.

3. ***H.C.F. of fractions***
H.C.F. = H.C.F. of numerators/L.C.M. of denominators

4. ***L.C.M. of fractions***
L.C.M. = L.C.M. of numerators/H.C.F. of denominators

5. If a,b,c are three numbers that divide a number n to leave the same remainder r, then the smallest value of n is
n = (L.C.M of a,b,c) + r

6. If a,b,c are three numbers that divide a number n to leave different remainders p, q, r respectively such that (a-p) = (b-q) = (c-r) = d, then the smallest value of n is
n = (L.C.M of a,b,c) - d

Ex. Find the H.C.F. of $14xy^3$, $26x^3y$ and $22x^2y$

Sol. $14xy^3 = 2 \times 7 \times x \times y^3$
$22x^2y = 2 \times 11 \times x^2 \times y$
$26x^3y^4 = 2 \times 13 \times x^3 \times y^4$
H.C.F. $= 2 \times x \times y$
$= 2xy$

Ex.. Find the L.C.M. of $10a^3b^2c^5$, $15a^2b^4c^3$ and $30a^3b^4c^5$.

Sol. $10a^3b^2c^5 = 5 \times 2 \times a^3 \times b^2 \times c^5$
$15a^2b^4c^3 = 5 \times 3 \times a^2 \times b^4 \times c^3$
$30a^3b^4c^5 = 5 \times 3 \times 2 \times a^3 \times b^4 \times c^5$
L.C.M. of the three numbers is: $5 \times 2 \times 3 \times a^3 \times b^4 \times c^5$
$= 30a^3b^4c^5$.

Ex. What is the least number of soldiers that can be drawn up in troops of 12, 18, 42 and 54 soldiers and also in the form of a solid square?

Sol. LCM of 12, 18, 42, 54 is 756. So these many students can be arranged in the toops of 12, 18, 42 and 54. To arrange in the form of a solid square, the number of soldiers should be a perfect square.
756 = 2 x 2 x 7 x 3 x 3 x 3
The given number becomes perfect square on multiplication by 3 x 7 i.e. 756 x 3 x 7 = 15876

Comparing Fractions: In order to compare fractions, reduce the given fractions to a common denominator (usually the L.C.M.) or common Numerator and compare the resulting numerators.
When denominators are equal, then the number with greater Numerator is greater.
When Numerators are equal, then the number with larger Denominator is smaller.

Exercise B

1. Find the H.C.F. for 1704, 3195 and 4970.

2. Find the H.C.F. of 35/12, 49/30 and 21/20.

3. Find the least number divisible by 12, 32, 42 and 63 and it must be a perfect square.

4. Find the least number which when doubled will be exactly divisible by 12,15,24, and 25.

5. The H.C.F. of two numbers is 113 and their L.C.M. is 228825. One of the numbers is 2825. Find the other.

6. If L.C.M. of two natural numbers is 36 and their sum is 30. Find the two numbers.

7. The sides of a triangular field are of lengths 2646, 5157 and 5634 metres. Find the greatest length of the tape by which the three sides may be measured completely.

8. Find the number between 2500 and 3000 which is divisible by 21, 24 and 28.

9. Find the least number divisible by each of the number 21,36,66. Which are the numbers less than 10,000 and divisible by 21, 36 and 66?

10. Three plots having an area of 132, 204 and 228 square metres respectively are to be sub-divided into equalized flowerbeds. If the breadth of a bed is 3 metres, find the maximum length that a bed can have.

11. There are 408 boys and 312 girls in a school which are to be divided into equal sections of either boys or girls alone. Find the maximum number of boys or girls that can be placed in a section. Also find the total number of sections thus formed.

12. A wine seller has three different types of wine, 403 gallon of 1st type, 434 gallon of 2nd type, 465 gallon of 3rd type. Find the least possible number of casks of equal size in which different types of wine can be filled without mixing.

13. How many square pieces (as large as possible) of identical dimensions can be cut from a rectangular field 200m long and 80m wide?

14. Four bells ring at intervals of 6,8,12 and 18 seconds. They start ringing simultaneously at 12 o' clock. Find when they will again ring simultaneously? How many times will they ring simultaneously within 6 minutes?

15. Three equal circular wheels revolve round a common horizontal axis with different velocities. The first makes a revolution is 5 1/3 minutes, the second in 2 6/7 minutes and the third in 3 3/7 minutes. Three markings, one in each wheel, are in a horizontal line at a certain moment. What is the shortest interval after which they will be in the same horizontal line again?

16. A, B, and C start at the same time from the same place in the same direction to walk round a circular course 12 miles long. A, B and C walk respectively at the rate of 3,7 and 13 miles per hour. In what time will they come together again at starting point?

17. The L.C.M. of two numbers is 28 times of their H.C.F. The sum of their L.C.M. and H.C.F. is 1740. If one of the numbers is 240, find the other number.

18. Find the side of largest possible square slabs which can be paved on the floor of a room 5 m 44 cm long and 3 m 74 cm broad. Also find the number of such slabs required to pave the floor.

19. The sum of two numbers is 684 and their H.C.F. if 57. Find all the possible pairs of such numbers.

20. Find the least number which will leave a remainder 9 when divided by 105, 112, 126, and 168.

21. Find the least number which when divided by 12, 16, 18, 30 leaves remainder 4 in each case but it is completely divisible by 7.

22. A heap of pebbles when made up into groups of 32, 40, 72 then the remainders are respectively 10, 18 and 50. Find the least number of pebbles in the heap.

23. Find the greatest number which will divide 1625, 2281 and 4218 leaving 8, 4 and 5 as remainders respectively.

24. Find the greatest number by which if we divide 166, 199 and 298, then in each case the remainder is the same.

25. What is the highest number of 4 digits which will leave a remainder of 1 when divided by any of the numbers 6,9,12,15 and 18?

26. Find the greatest number of 5 digits which when divided by 8, 12, 15 and 20 leaves respectively 5,9,12 and 17 as remainders.

27. On 26th January 1986, students of a school were made to stand in several rows. Each row had as many students as were the total number of rows. If the total number of students was 1024, how many students were standing in each row?

28. Find H.C.F. and L.C.M. of the following
(a) $4ab^2c^3d$, $6a^3bc^4d^2$, $10ab^4d$ (b) $8x^3y^4z^6$, $12xy^5z^2$ (c) a^3b^2, a^4b, a^2y^3z
(d) $a^2 + ab$, $a^2 - b^2$ (e) $(x + y)^2$, $x^2 - y^2$ (f) $x(a - x)^2$, $a(a - x)^3$
(g) $2x^2 - 2xy$, $x^3 - x^2y$ (h) $a^2 + ab$, $ab + b^2$

29. A gardener planted 103041 trees in such a way that the number of rows were as many as were the trees in a row find the number of rows.

30. A man after completing his tour found that he has expensed the same money in Rs. per day as the number of days he was on tour. If he expensed Rs.7744 in total, find out for how many days he was on tour?

31. The soldiers of an army were made to stand in a solid square. In doing so, 5 soldiers were left. If there were only 5630 soldiers in the army, find the number of soldiers in the outermost row of the solid square.

32. Find the least number which when added to or subtracted from 1850 makes it a perfect square.

33. Find the least number by which if we multiply 11520, the product becomes a perfect square.

34. Find the least number by which if we divide 2816, the quotient may be a perfect square.

35. Find the greatest number of 6 digits which is a perfect square.

36. A General can draw up his soldiers in the rows of 10, 15 or 18 soldiers and he can also draw them up in the form of a solid square. Find the least number of soldiers with the general.

37. If the sum of two numbers be multiplied by each separately, the products so obtained are 2418 and 3666. Find the numbers.

38. Find the smallest number by which 1715 be multiplied to make it a perfect cube. What is the perfect cube number? Find its cube root.

39. There are few students in a class. Each student gave as many paise as the contribution to the flood relief fund as was the square of the number of students. The total contribution was Rs.297.91. Find the number of students in the class.

40. Some packets of chalk are kept in a box in the form of a cube. If the total number of packets is 2197, find the no. of rows of the packets in each layer.

Answers

1. 71	**2.** 7/60	**3.** 28224	**4.** 300	**5.** 9153
6. 12, 18	**7.** 9 metres	**8.** 2520, 2688, 2856	**9.** 2772, 5544, 8316	
10. 4 metres	**11.** 24,30	**12.** 42	**13.** 10 or 2	
14. 72 secs., 5 times		**15.** 4 hr.	**16.** 12 hr.	**17.** 420
18. 34 cms,176	**19.** 57,627 & 285,399	**20.** 5049	**21.** 2884	**22.** 1418
23. 11	**24.** 33	**25.** 9901	**26.** 99957	**27.** 32
28. (a) 2abd, $60a^3b^4c^4d^2$		**(b)** $4xy^4z^2$, $24x^3y^5z^6$	**(c)** a^2, $a^4b^2y^3z$	
(d) a +b, a^3 - ab^2	**(e)** x + y, $(x + y)^2$ (x - y)	**(f)** $(a - x)^2$, ax $(a - x)^3$	**(g)** x^2 - xy, $2x^3$ - $2x^2y$	
(h) a + b, ab (a + b)		**29.** 321	**30.** 88	
31. 75	**32.** 1 (to be subtracted)	**33.** 5	**34.** 11	**35.** 998001
36. 900	**37.** 31, 47	**38.** 25, 42875,35	**39.** 31	**40.** 13

INDICES, SQUARE ROOTS & CUBE ROOTS

$a^n = a \times a \times a \ldots$ n times. Here, a is called the base and n is called the index or the power or the exponent.

Basic laws of indices :

$a^m \times a^n = a^{m+n}$

$a^m/a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$\sqrt{a} = a^{(1/2)}$

$a^1 = a$

$a^n b^n = (ab)^n$

$a^n/b^n = (a/b)^n$

$a^{-n} = 1/a^n = (1/a)^n$

$\sqrt[n]{a} = a^{(1/n)}$

$a^0 = 1$, if $a \neq 0$.

Formulae To Be Remembered

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^2 - (a - b)^2 = 4ab$
4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
5. $(a + b)(a - b) = a^2 - b^2$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + 3ab(a + b) + b^3$
   $\therefore a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ or $(a+b)(a^2+b^2-ab)$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - 3ab(a - b) - b^3$
   $\therefore a^3 - b^3 = (a - b)^3 + 3ab(a - b)$ or $(a-b)(a^2+b^2+ab)$
8. $(x + a)(x + b) = x^2 + (a + b)x + ab$.
9. $(x - a)(x + b) = x^2 + (b-a)x - ab$.
10. $(x - a)(x - b) = x^2 - (a+b)x + ab$.
11. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
12. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3ab(a + b) + 3bc(b + c) + 3ac(a + c) + 6abc$.
    $= a^3 + b^3 + c^3 + 3(a+b)(b + c)(c + a)$.
    $= a^3 + b^3 + c^3 + 3(a+b+c)(ab+bc+ac) - 3abc$.
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$.
14. $n! = 1 \times 2 \times 3 \times \ldots \times (n - 2) \times (n - 1) \times n$. (Where n is a natural number.)
15. $0! = 1$.

Additional Rules on Numbers

1. If n is an odd number greater than 1 then $n(n^2 - 1)$ is divisible by 24.
2. If n is an odd prime number greater than 3 then $(n^2 - 1)$ is divisible by 24.
3. If n is an odd number, then $(2^n + 1)$ is divisible by 3.
4. If n is an even number, then $(2^n - 1)$ is divisible by 3.
5. If n is an odd number, then $(2^{2n} + 1)$ is divisible by 5.
6. If n is an even number, then $(2^{2n} - 1)$ is divisible by 5.
7. If n is an even number, then $(2^{2n} - 1)$ is divisible by 15.
8. If n is an odd number, then $(5^{2n} + 1)$ is divisible by 13.
9. If n is an even number, then $(5^{2n} - 1)$ is divisible by 13.
10. If n is any natural number, then $(5^{2n} - 1)$ is divisible by 24.
11. If n is co-prime to 5, then $n(n^4 - 1)$ is divisible by 30.
12. $(x^n + y^n)$ is divisible by $(x + y)$ when n is an odd number.

13. $(x^n - y^n)$ is divisible by $(x + y)$ when n is an even number.
14. $(x^n - y^n)$ is divisible by $(x - y)$ when n is an odd or an even number.
15. i) If p is a prime number then for any whole number a, $a^p - a$ is divisible by p.
ii) 1 is not a prime number.

Ex. Find 380^2

Sol. $380^2 = (400 - 20)^2$
$= 160000 - 16000 + 400$
$= 144000 + 400$
$= 144400$

Ex. Given that $x^2 + 1/x^2 = 47$. Find $x + 1/x$.

Sol. $(x + 1/x)^2 - 2 = x^2 + 1/x^2$
$\therefore (x + 1/x)^2 = 47 + 2 = 49$
$(x + 1/x) = \pm 7$

Ex. Find $a^2 + b^2$ if $(a + b) = 25$ and $ab = 150$. Also find $a^3 + b^3$

Sol. $a^2 + b^2 = (a + b)^2 - 2ab$
$\therefore a^2 + b^2 = 625 - 300$
$= 325$

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
$\therefore a^3 + b^3 = 15625 - 450\ (25)$
$a^3 + b^3 = 15625 - 11250$
$= 4375$

Ex. Simplify $\dfrac{(6.4)^3 + (5.6)^3}{(6.4)^2 - (5.6)^2}$

Sol. Let a = 6.4 and b = 5.6. The given sum is of the form:
$$\frac{a^3 + b^3}{a^2 - b^2} = \frac{(a+b)\ (a^2 - ab + b^2)}{(a+b)\ (a - b)} = \frac{(a^2 - ab + b^2)}{(a - b)} = \frac{(6.4)^2 - (6.4 \times 5.6) + (5.6)^2}{6.4 - 5.6} = 45.6.$$

Square Root and Cube Root

Square Root

Square Root of a given number is that number which on multiplication by itself results in the given number. e.g. if y = x*x, then x is called as the square root of y.

Cube Root

Cube Root of a given number is that number whose cube is the given number. e.g. if y = x*x*x, then x is called as the cube root of y.

Square Root by the methods of factors

Resolve the given number into prime factors and make pairs of similar factors. If all the factors exist in pairs then the given number is a perfect square. To find the square root of the given number, take the product of prime factors, choosing one out of every pair.

Ex. Find the square root of 3969
Sol. $3969 = 3 \times 3 \times 3 \times 3 \times 7 \times 7$ $\therefore$ square root of $3969 = 3 \times 3 \times 7 = 63$

Ex. Find the square root of 530.3809.
Sol. Square root to the given number can be found by long division method of finding square roots. It is quite similar to the simple division. The method is explained below:

	2 3 . 0 3
2	5 3 0 . 3 8 0 9
+ 2	4
4 3	1 3 0
+ 3	1 2 9
4 6 0	1 3 8
+ 0	0 0 0
4 6 0 3	1 3 8 0 9
	1 3 8 0 9
	0 0 0

- Make pairs of the numbers starting from the decimal. For the numbers on the LHS of the decimal, make pairs going right to left. For the numbers on the RHS of decimal, make pairs left to right. It does not matter if a number on the LHS of decimal remains single. Here the pairs are (5)(30).(38)(09)
- Look for the perfect square smaller than or equal to the first pair of single digit as applicable. Here the nearest perfect square to 5 is $4 = 2^2$.
- Square root of this perfect square becomes the first divisor. So 2 will be divisor as well as the first digit of the dividend. The final value of quotient is the square root of the number.
- Divide as in normal division. So we get 1 as remainder in this case.
- Carry down the next pair. Here 30
- Form one part of the new divisor by adding to the divisor its unit digit. Here 2 + 2 = 4. The other part of the divisor is its unit digit which is decided depending upon the dividend. Here we need to divide 130 and the divisor is a two-digit number starting with 4. So the highest unit digit possible is 3. Write this 3 as the next digit of the quotient.
- Again carry down the next pair. Because there is decimal before this pair, put decimal in the quotient as well. Add 3 to 43 to get a part of the next divisor.
- Next three-digit divisor 46_ is greater than the dividend 138 we make the divisor as 460 and one zero is added to the quotient.
- Carry down the next pair. So the dividend becomes 13809 and the divisor is 460_. Put 3 as the unit digit to find that 4603*3 = 13809 and hence the division terminates. Write 3 as the last digit of quotient.

Square root of 530.3809 is 23.03

Ex. What is the cost of erecting a fence round a square field of area 1000000 square metres at the rate of Rs. 10 per metre?

Sol. Area of the field = 1000000 sq. m.
$\therefore$ side of the field = $\sqrt{1000000}$ = 1000 m
$\therefore$ perimeter of the field = 4 x 1000 = 4000 m
$\therefore$ cost = 10 x 4000 = Rs. 40000

Ex. Find the least number that is a perfect square and which is also divisible by 16, 18 & 45.

Sol. L.C.M. of 16, 18 & 45 = 720
Multiplying 720 by 5 we get the required number = 720 x 5 = 3600

Cube Root by the method of factors

Resolve the given number into prime factors and choose one out of every three of the same type.

Ex. Find the cube root of 343.

Sol. 343 = 7 x 7 x 7. $\therefore$ cube root of 343 = 7

Ex. By what number should 21600 be multiplied to make it a perfect cube?

Sol. $21600 = 2^3 \times 2^2 \times 3^3 \times 5^2$
Thus 21600 should be multiplied by 2 x 5 to make it a perfect cube
$\therefore$ The perfect cube number = 216000
Cube root of 216000 = 2 x 2 x 3 x 5 = 60

POWER CYCLES

2, 4, 8, 16, 32, 64, 128, 256, 512, …… The series is nothing but powers of 2 in ascending order.

if you see it carefully we see that there is repitition of unit digit after regular intervals or unit digits always have definite pattern. The unit digit always follows a cycle which is termed as power cycles.

This cycle is followed by each and every number although the frequency of repitition of cycle is not same for e.g power cycle of 2 has frequency of 4 where as 5 has a frequency of 1. Let us examine the power cycle of each number

1 : every time the number in the unit digit will be always 1
2 : 2, 4 , 8, 6, after this again repetition of cycle starts .
3 : 3, 9, 7, 1
4 : 4, 6
5 : 5
6 : 6
7 : 7, 9, 3, 1
8 : 8, 4, 2, 6
9 : 9, 1

Let us say if question is asked what is digit at units place in 7^{71} as 7 has power cycle with frequency 4 when we divide 71 by 4 the remainder is 3 and the third power of 7 has unit digit as 3, which is the answer

EXERCISE C

1. What is x if $0^X = 0$

2. Which is greater $(2.3)^{0.6}$ or $(2.3)^{0.65}$

3. Solve $\sqrt[3]{(27a^3b^9c^{15})}$

4. Which is greater 2^{400} or 4^{200} ?

5. $2^{x+3}.4^x = 512$. Find x

6. Simplify $3\sqrt{18}+2\sqrt{8}+3\sqrt{32}-2\sqrt{50}$.

7. Find x if $2^{x-2} + 4 \times 5^{x-5} = 132$

8. Simplify: (a) a^{-7}/a^{-5} (b) $[\sqrt[7]{x^2\sqrt{x^3}}]$

9. Find x if $[2^{x-1} \times 4^{2x+1}]/[8^{x-1}] = 64$

10. A number when divided by 18 gives a remainder 13. What is the remainder obtained by dividing the same number with 9?

11. Find the least number, which must be added to 36520 to make it exactly divisible by 187.

12. Is 467016 divisible by 132?

13. Find the number, which is nearest to 17289 and is exactly divisible by 193.

14. Find the value of 511^2.

15. Find the value of 781 x 819

16. If $x^2 + 1/x^2 = 18$, find $x - 1/x$

17. Find $p^2 + q^2$ if $p - q = 5$ and $pq = 66$

18. Find $x^3 + y^3 + z^3 - 3xyz$, when $x + y + z = 9$ and $xy + yz + zx = 11$

19. If X = b+ c - 2a, Y = c + a - 2b, and Z = a + b - 2c, find the value of $X^3 + Y^3 + Z^3 - 3XYZ$.

20. If $\frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy} = 3$, find x + y + z.

21. If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$, and a + b + c = 5, find the value of $a^3 + b^3 + c^3 - 3abc$.

22. (x-a)(x-b)(x-c).....(x-z) = ?

23. If a = b = c = d, prove that $a^2 + b^2 + c^2+ d^2 = ab + bc + cd + da$.

24. If a = b = c = d, prove that $(a + b)^2 + (b + c)^2 + (c+ d)^2 = 4(ab + bc + cd)$.

25. (!) What will be the unit digit of 3564328^{161}?
(!!) What will be the unit digit of $61^{91} + 345^{201} - 176^{46}$?
(!!!) What will be the remainder when $(7^{48} - 11^7)$ is divided 5?

26. If $a + b + c = 0$, is $a^3 + b^3 + c^3$ an even number, where a, b and c are three distinct integers?

27. Find the real cube root of $(9\sqrt{3} + 11\sqrt{2})$.

28. Prove that $(20 + 14\sqrt{2})^{1/3} + (20 - 14\sqrt{2})^{1/3} = 4$.

29. If $A/a = B/b = C/c = D/d$, then prove that
$(Aa)^{1/2} + (Bb)^{1/2} + (Cc)^{1/2} + (Dd)^{1/2} = [(a + b + c + d)(A + B + C + D)]^{1/2}$.

30. Supply the missing digit that makes 8276H845, divisible by 11.

31. $\left(\dfrac{11.347\times11.347\times11.347 + 18.653\times18.653\times18.653}{11.347\times11.347 + 18.653\times18.653 - 11.347\times18.653}\right) = ?$

32. Evaluate $(97 + 56\sqrt{3})^{1/4}$.

33. Simplify $[(256)^{6.25} \times (16)^{2.7}] / [(256)^{6.75} \times (16)^{1.2}]$

34. Simplify $\dfrac{26.21 \times 26.21 - 14.79 \times 14.79}{4.1 \times 26.21 - 4.1 \times 14.79}$

35. Find the square root of 0.7.

36. Simplify $\dfrac{(0.023)^3 + (0.117)^3}{0.023 \times 0.023 - 2.3 \times 0.00117 + 1.17 \times 0.0117}$

37. Simplify $\dfrac{0.07 \times 0.07 - 0.07 \times 0.03 + 0.03 \times 0.03}{0.07 \times 0.07 \times 0.07 + 0.03 \times 0.03 \times 0.03}$

38. Simplify $\dfrac{0.6 \times 0.6 \times 0.6 + 0.4 \times 0.4 \times 0.4}{0.6 \times 0.6 - 0.6 \times 0.4 + 0.4 \times 0.4} + \dfrac{0.5 \times 0.5 + 0.4 \times 0.4 + 2 \times 0.5 \times 0.4}{0.5 \times 0.5 - 0.4 \times 0.4}$

39. Simplify $\dfrac{43.5\text{x}43.5 + 33.5\text{x}33.5 + 2\text{x}33.5\text{x}43.5}{43.5\text{x}43.5 - 33.5\text{x}33.5} + \dfrac{16.5\text{x}16.5 + 6.5\text{x}6.5 + 2\text{x}6.5\text{x}16.5}{16.5\text{x}16.5 - 6.5\text{x}6.5}$

40. Simplify $\dfrac{1.01\text{x}1.01\text{x}1.01 + 0.01\text{x}0.01\text{x}0.01}{1.01\text{x}1.01 - 0.01\text{x}0.01} - \dfrac{0.99\text{x}0.99\text{x}0.99 - 0.01\text{x}0.01\text{x}0.01}{0.99\text{x}0.99 - 0.01\text{x}0.01}$

41. Simplify $\dfrac{(7.2)^3 - (2.8)^3}{(9.1)^3 - (4.7)^3} \times \dfrac{(9.1)^2 + (4.7)^2 + 10\text{x}0.91\text{x}4.7}{(7.2)^2 + 196\text{x}0.04 + 63\text{x}0.8\text{x}0.4}$

42. Simplify $\dfrac{7.85\text{x}7.85 - 2.15\text{x}2.15}{7.85 - 2.15} - \dfrac{(1/3) + (1/5) \div (1/7)}{(1/9) \div \{(1/2) + (1/13)\}}$

43. Simplify $\dfrac{0.2 \times 0.4 \times 0.1 + 0.03 \times 0.9}{0.04 - 0.06 + 0.09}$

44. What will be the remainder obtained when $(9^6 + 1)$ is divided by 8?

45. What is the highest power of 3 that will exactly divide 100! .

46. Find the last digit of the number $(7)^{71}$?

47. Which of the following is greater 2^{300}, 3^{200} or 5^{150} ?

48. How many zeros are there at the end of the decimal representation of the number 100! ?

49. Find the remainder when the number 7^{100} is divided by 6?

50. If x is a single digit natural number, find the digit at the unit's place of x^5?

ANSWERS

1. Any non-zero value.	**2.** $(2.3)^{0.65}$	**3**. $3ab^3c^5$	**4.** Both are equal
5. 2	**6.** $15\sqrt{2}$	**7**. 7	**8. (a)**.($1/a^2$) **(b)**. $\sqrt{x}$
9. 1	**10**. 4	**11**. 132	**12**. Divisible
13. 17370	**14**. 261121	**15**. 639639	**16**. ± 4
17. 157	**18**. 432	**19**. 0	**20**. Can't be determined
21. 125	**22**. 0.	**25**. (!) 8, (!!) 0, (!!!) 0	**26**.Even
27. $\sqrt{3} + \sqrt{2}$	**30**. 2	**31**. 30	**32**. $2 + \sqrt{3}$.
33. ± 4.	**34**. 10	**35.** 0.8366	**36**. 0.14
37. 10	**38.** 10	**39.** 10	**40**. 0.02
41. 1	**42**. 1	**43.** 0.5	**44.** 2
45. 48	**46.** 3	**47**. 5^{150}	**48.** 24
49. 1	**50.** x itself		

Solutions

Exercise A

Que. 1 to 10 : These questions are based on simple mathematical operations like addition, subtraction, multiplication & division and can be easily solved using BODMAS. Answers are already given just below the exercise.

11. **(i)** X381 is the number given to us. If this number is divisible by 11 then the divisibility rule for 11 must be satisfied i.e. $(X + 8) – (3 + 1) = 0$ or a multiple of 11
i.e. $X + 4 = 11 \Rightarrow X = 7$
(ii) 381Y is the number given to us. If this number is divisible by 9 then the divisibility rule for 9 must be satisfied i.e. $(3 + 8 + 1 + Y)$ must be divisible by 9 i.e. $(12 + Y) = 18 \Rightarrow Y = 6$

12. (1234567890123456789) is the number given to us. Sum of all the digits = 2(1+ 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 2 (45) = 90 i.e. this number is divisible by 9. Thus we can write $(1234567890123456789)^{24}$ as $(9K)^{24}$. Similarly 6561 can be written as 9^4. This means that the given number will be completely divisible by 6561 and remainder will be 0.

13. Since $990 = 2 \times 3^2 \times 5 \times 11$, n! must contain a factor of 11. Since 11 is a prime number, it must itself be contained in the product, so n = 11 is the smallest possible value.

14. The specialty of number 9 is that whatever the number be divided by 9, first you have to take the sum of the digits and it should go on till the sum can be written as single digit, which will be the reminder. For e.g., taking 1987, first sum which we have 1 + 9 + 8 + 7 = 25. Again 2 + 5 = 7, so 7 will be the reminder. In case if last sum comes out to be 9 then 0 will be the reminder. In the same manner, for 12345678910111213……..24, the sum of the individual digits comes out to be 48 and again the sum comes out to be 12 at last which gives 3. So the reminder will be 3.

15. r_1, r_2, r_3, r_4 be the reminders when 1237, 1239, 1241, 1243 divided by 30 respectively . The reminder will be the reminder which we will have by dividing the product of reminders with the number given, 7, 9, 11, 13 are the reminders and when 7 x 9 x 11 x 13 divided by 30 will have 9 as reminder.

16. Equating the denominators, we get,
3/7 is less then 1/2, $\therefore$ We should compare only last two fractions.
L.C.M. of 9 and 12 is 36.
5/9 = 20/36 and 7/12 = 21/36.
$\therefore$ 7/12 > 5/9 > 3/7.
7/12 is greater than all other fractions.

17. Taking the L.C.M. of the denominators, we can convert the given fractions as 615/ 780, 640/ 780 and 648/ 780. So the fraction 41/ 52 is the smallest.

18. $a^n / (a + 1) = [(a + 1) – 1]^n / (a + 1)$ On expanding this expression, we get all the terms but for one having a factor of (a + 1). So it becomes $\{k(a + 1) + (-1)^n\} / (a + 1)$ where k is a whole number. The term which decides remainder is $(-1)^n\}/ (a + 1)$
(a) When n is even this term becomes 1/(a + 1)…and hence answer 1.
(b) When n is odd this term becomes -1/(a + 1)…and hence remainder is –1 or a + 1 – 1 = a.

19. Here the number of digits is 20, which is even. So D = (5 + 5 + 5 + …10 times) – (8 + 8 + 8 + …10 times) = 50 – 80 = - 30. If we divide -30 by 11, the remainder obtained is -8 which is smaller than 0. So answer is 11 – 8 = 3.

20. In this question to compare the highest powers of 5 and 7, let us start with any random value say 35. Look at the table…you'll find that the least no. which has difference of 7 is 75 and then there are several other numbers which have the difference of 7.

NO.	Power of 5	Power of 7	Difference
35	8	5	3
70	16	11	5
75	18	11	7
76	18	11	7
77	18	12	6
78	18	12	6
79	18	12	6
80	19	12	7
81	19	12	7
82	19	12	7
83	19	12	7
84	19	13	6

Exercise B

1. 1704 = 2 x 2 x 2 x 3 x 71
   3195 = 3 x 3 x 5 x 71
   4970 = 2 x 2 x 5 x 7 x 71
   HCF = 71

2. HCF of fractions = HCF of numerators/LCM of denominators.
   35 = 5 x 7, 49 = 7 x 7 and 21 = 3 x 7.
   The HCF of 35, 49 and 21 is 7.
   12 = 2 x 2 x 3, 30 = 2 x 3 x 5 and 20 = 2 x 2 x 5.
   LCM of 12, 30, 20 = 2 x 2 x 3 x 5 = 60.
   Therefore, the HCF of the given fractions is 7/60.

3. LCM of 12, 32, 42, 63 = 2016
   2016 = 2 x 2 x 2 x 2 x 2 x 3 x 3 x 7
   The perfect square, which is a multiple of all the given numbers, is
   2016 x 2 x 7 = 28224

4. Required number = (1/2) x LCM of given numbers

2	12	15	24	25
2	6	15	12	25
2	3	15	6	25
3	3	15	3	25
5	1	5	1	25
5	1	1	1	5
	1	1	1	1

   LCM = 2 x 2 x 2 x 3 x 5 x 5 = 600. Therefore, the required number is 300.

5. 228825 = 113(f_1 x f_2).
   2825 = 113 x f_1.
   f_1 = 25 and f_2 = 81. Therefore, the other number is 113 x 81 = 9153.

6. The two natural numbers could be any two of the factors of 36 viz. 1,2,3,4,6,9,12,18.
   But only 12 + 18 = 30. Hence the required numbers are 12 and 18.

7. Find the HCF of the given numbers
   2646 = 2 x 3 x 3 x 3 x 7 x 7
   5157 = 3 x 3 x 3 x 191
   5634 = 2 x 3 x 3 x 313
   HCF = 3 x 3 = 9; Length of the tape = 9 metres

8. LCM of 21, 24, 28
   21 = 3 x 7
   24 = 2 x 2 x 2 x 3
   28 = 2 x 2 x 7
   LCM = 2 x 2 x 2 x 3 x 7 = 168
   Multiples of 168 between 2500 & 3000 are 2520, 2688, 2856

9. LCM of 21, 36, 66

2	21	36	66
2	21	18	33
3	21	9	11
3	7	3	11
7	7	1	11
11	1	1	11
	1	1	1

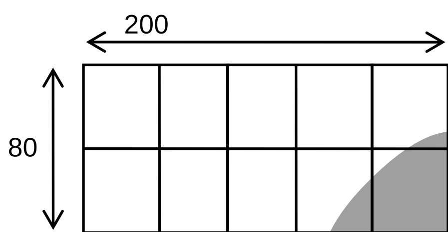
LCM = 2 x 2 x 3 x 3 x 7 x 11 = 2772
Multiples of 2772 less than 10000 are: 2772, 5544, 8316

10. HCF of 132, 204, 228 = 2 x 2 x 3 = 12
Area of bed = 12 sq. m.
Length of bed = 12/3 = 4 meters.

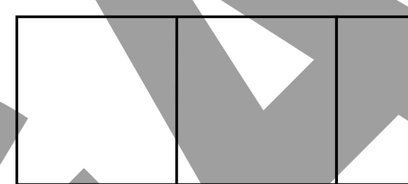
11. HCF of 408 & 312 = 24
maximum no. of students in a group = 24
Total number of sections = (408 + 312)/24 = 30

12. HCF of 434, 465 is 31
wine 1 can be stored in 13 casks of 31 gallon size
wine 2 can be stored in 14 casks of 31 gallon size
wine 3 can be stored in 15 casks of 31 gallon size
Total no, of casks = 13 + 14 + 15 = 42

13.



(a)



(b)

If the entire field is to be utilised, the solution will be as shown in (a) above with 10 pieces of 40x40. If the above condition is not mandatory, 2 pieces of dim. 80x80 can be cut with a strip of field of dim. 40x80 remaining unutilised, as shown in fig.(b) above.

14. LCM of 6, 8, 12, 18 is 72
The bells will ring again simultaneously after 72 seconds i.e. 1 min. 12 seconds
In six minutes they will ring simultaneously (6)(60)/72 = 5 times

15. Wheel 1 = 16/3 minutes
Wheel 2 = 20/7 minutes
Wheel 3 = 24/7 minutes
LCM of 16/3, 20/7 , 24/7 = 240
The markings will come together in 240 min. = 4 hours

16. Proceed as in Problem No. 15.

17. L + H = 1740. L = 28H.
H = 60 and L = 1680
$1680 = 60(f1 \times f_2)$
240 = 60 x f_1. Therefore, $f_1 = 4$ and $f_2 = 7$. Hence, the other number is 60 x 7 = 420.

18. Find HCF of 544 & 374
544 = 2 x 2 x 2 x 2 x 2 x 17
374 = 2 x 11 x 17
HCF = 2 x 17 = 34
Side of the tile = 34 cm
No. of tiles required. = (544 x 374)/(34 x 34) = 176

19. $684 = 57(f_1 + f_2)$
$f_1 + f_2 = 12$, and f_1, f_2 are relatively prime. So, $f_1 = 1$ $f_2 = 11$ OR $f_1 = 5$ $f_2 = 7$.
Therefore, the required numbers are 57 and 627 OR 285 and 399.

20. The required number = LCM + 9
$105 = 3 \times 5 \times 7$
$112 = 2 \times 2 \times 2 \times 2 \times 7$
$126 = 2 \times 3 \times 3 \times 7$
$168 = 2 \times 2 \times 2 \times 3 \times 7$
LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 5040$
The required number = 5049

21. LCM of 12, 16, 18, 30 = 720
Required Number = 720y + 4 (divisible by 7)
$720y = 7(102y) + 6y + 4$.
$y = 4$
Required Number = $720 \times 4 + 4 = 2884$

22. Required Number = LCM – d, where d = 32 - 10 = 40 - 18 = 72 - 50 = 22
LCM of 32, 40, 72 = 1440
Required Number = 1440 - 22 = 1418

23. Required Number = HCF of
$1625 - 8 = 1617 = 11 \times 147$
$2281 - 4 = 2277 = 3 \times 3 \times 11 \times 23$
$4218 - 5 = 4213 = 11 \times 383$
HCF = 11

24. Let remainder = r
A = 166 - r, B = 199 - r, C = 298 - r
A, B & C should be divisible by the common divisor. Since it is the largest such number, it must be the H.C.F. of the above numbers. If A, B, C are divisible by this number then B - A & C - B must also be divisible by this number.
B - A = 199 - r - 166 + r = 33
C - B = 298 - r - 199 + r = 99
33 & 99 must be divisible by this number. $\therefore$ the required number is 33
(Cross check :166, 199, 298 when divided by 33 leave a remainder = 1)

25. LCM of 6, 9, 12, 15, 18 is 180
4 digit multiple of 180 = $180 \times 55 = 9900$
Required Number = 9900 + 1 = 9901

26. LCM of 8, 12, 15, 20 is 120
5 digit multiple of 120 = 99960
Required Number = 99960 – d, where, d = 8 - 5 = 12 - 9 = 15 - 12 = 20 - 17 = 3
Required Number = 99960 - 3 = 99957.

27. Let the number of rows be x, number of students in each row be x.
$(x)(x) = x^2 = 1024 = 4 \times 256$
$\therefore x = 32$

28.
(a) $4ab^2c^3d = 2 \times 2 \times a \times b^2 \times c^3 \times d$
$6a^3bc^4d^2 = 2 \times 3 \times a^3 \times b \times c^4 \times d^2$
$10ab^4d = 2 \times 5 \times a \times b^4 \times d$
$\therefore$ HCF $= 2 \times a \times b \times d = 2abd$
$\therefore$ LCM $= 2 \times 2 \times 3 \times 5 \times a^3 \times b^4 \times c^4 \times d^2 = 60a^3b^4c^4d^2$

(b) $8x^3y^4z^6 = 2 \times 2 \times 2 \times x^3 \times y \times z^6$
$12xy^5z^2 = 2 \times 2 \times 3 \times x \times x \times y^5 \times z^2$
$\therefore$ HCF $= 2 \times 2 \times x \times y^4 \times z^2 = 4xy^4z^2$
$\therefore$ LCM $= 2 \times 2 \times 2 \times 3 \times x^3 \times y^5 \times z^6 = 24x^3y^5z^6$

(c) $a^3b^2 = a \times a \times a \times b \times b$
$a^4b = a \times a \times a \times a \times b$

$a^2y^3z = a \times a \times y \times y \times y \times z$
$\therefore$ HCF $= a \times a = a^2$
$\therefore$ LCM $= a \times a \times a \times a \times b \times b \times y \times y \times y \times z = a^4b^2y^3z$

(d) $a^2 + ab = a \times (a + b)$, and $a^2 - b^2 = (a + b) \times (a - b)$
The common factor is $(a + b)$. The H.C.F. is $(a + b)$.
The L.C.M. is $a \times (a + b) \times (a - b) = a \times (a^2 - b^2) = a^3 - ab^2$

(e). $(x + y)^2 = (x + y)(x + y)$, $x^2 - y^2 = (x + y)(x - y)$
H.C.F. $= (x + y)$
L.C.M. $= (x + y)(x + y)(x - y)$

(f) $x(a - x)^2 = x(a - x)(a - x)$, $a(a - x)^3 = a(a - x)(a-x)(a-x)$
H.C.F. $= (a - x)(a - x)$
L.C.M. $= a.x.(a - x)(a-x)(a - x) = ax(a - x)^3$

(g) $2x^2 - 2xy = 2.x.(x - y)$, $x^3 - x^2y = x.x.(x - y)$
H.C.F. $= x.(x - y) = x^2 - xy$
L.C.M. $= 2.x.x.(x - y) = 2x^3 - 2x^2y$

(h) $a^2 + ab = a.(a + b)$, $ab + b^2 = b.(a + b)$
H.C.F. $= (a + b)$
L.C.M. $= a.b.(a + b)$

29. Let the number of rows be x, number of trees in each row be x.
$(x)(x) = x^2 = 103041$
$x = 321$

30. Let the number of days be x, money spent on each day be Rs. x
Total expenditure $= (x)(x) = x^2 = 7744 = 11 \times 11 \times 64$
$x = 88$

31. No. of soldiers in the square $= 5630 - 5 = 5625$
Let the number of rows be x, number of soldiers in each row = x
$(x)(x) = x^2 = 5625 = 25 \times 225$
$\therefore x = 5 \times 15 = 75$

32. 1850 is (1849 + 1) i.e. $(43)^2 + 1$ and also (1936 -86) i.e. $(44)^2 - 86$. So, if 1 is subtracted from 1850 or if 86 is added to 1850, we get perfect squares. So, the least number to be added or subtracted to get a perfect square is 1.

33. $11520 = 16 \times 16 \times 3 \times 3 \times 5$
If 11520 is multiplied by 5 it becomes a perfect square whose square root is $(16 \times 3 \times 5) = 240$

34. $2816 = 16 \times 16 \times 11$
If 2816 is divided by 11 it becomes a perfect square

35. The least number of seven digits which is a perfect square is 1000000, whose square root is 1000. The greatest number of six digits which is a perfect square is the square of 999 i.e. 998001

36. The L.C.M. of 10,15,18 is 90
Thus, the perfect square, which is divisible by all three, is 900
The number of soldiers is 900

37. Let the numbers be x & y
$x(x + y) = 2418$ and $y(x + y) = 3666$
Solving these two equations simultaneously,
$x = 31$ $y = 47$

38. $1715 = 5 \times 7 \times 7 \times 7$
$1715 \times 5 \times 5 = 5 \times 5 \times 5 \times 7 \times 7 \times 7 = 42875$
1715 should be multiplied by 25 to make it a perfect cube : 42875
Cube root of $42875 = 5 \times 7 = 35$

39. Let the number of students in the class be x. Each one contributes x^2 paise
$x(x^2) = 29791$
$x = 31$

40. Let the number of rows be x. Then, $x^3 = 2197$ or $x = 13$.

Exercise C

1. For any x, $a^x = 0$, if and only if $a = 0$. and $x \neq 0$.
   Hence, x can take any value, real or imaginary; other than 0. As 0^0 is not defined.

2. Since 2.3 > 1 and 0.60 < 0.65, It is clear that
   $(2.3)^{0.6} < (2.3)^{0.65}$

3. $(27a^3b^9c^{15})^{1/3} = (27^{1/3})(a^{3/3})(b^{9/3})(c^{15/3})$
   $= 3ab^3c^5$

4. $4^{200} = (2^2)^{200} = 2^{400}$
   $\therefore 2^{400} = 4^{200}$

5. $2^{x+3} . 4^x = 512$
   $\therefore$ L.H.S. $= 2^{x+3}.(2^2)^x = 2^{x+3}.2^{2x}$
   $= 2^{3x+3}$
   R.H.S. $= 512 = 2^9$

   $\therefore 2^{3x+3} = 2^9$
   $\Rightarrow 3x+3 = 9$
   Hence, $x = 2$.

6. $3\sqrt{18} + 2\sqrt{8} + 3\sqrt{32} - 2\sqrt{50}$
   $= 3\sqrt{3\times3\times2} + 2\sqrt{2\times2\times2} + 3\sqrt{4\times4\times2} - 2\sqrt{5\times5\times2}$
   $= 9\sqrt{2} + 4\sqrt{2} + 12\sqrt{2} - 10\sqrt{2}$
   $= 15\sqrt{2}$.

7. $2^{x-2} + 4(5^{x-5}) = 132$
   $\therefore 4.(2^{x-4} + 5^{x-5}) = 132$
   $2^{x-4} + 5^{x-5} = 33$
   Since both terms on the L.H.S are +ve, maximum value of x - 5 is 2.
   i.e. max. value of x is 7.
   We try and eliminate values of 6 and 8 for x.
   Hence, x = 7.
   (For any $x \leq 0$, max. value of L.H.S. is always less than 2).

8a. $a^{-7}/a^{-5} = a^{-7+5} = a^{-2} = 1/a^2$

8b. $\sqrt[7]{x^2\sqrt{x^3}} = \{x^2(x^3)^{1/2}\}^{1/7}$
   $= \{x^{2+3/2}\}^{1/7}$
   $= (x^{7/2})^{1/7}$
   $= x^{1/2} = \sqrt{x}$

9. $[2^{x-1} . 4^{2x+1}] / 8^{x-1} = 64$

   L.H.S. $= [2^{x-1} . (2^2)^{2x+1}] / (2^3)^{x-1} = [2^{x-1} . 2^{4x+2}] / 2^{3x-3} = 2^{5x+1} / 2^{3x-3} = 2^{2x+4}$
   R.H.S. $= 64 = 2^6$
   $\therefore 2^{2x+4} = 2^6$
   $\therefore 2x+4 = 6$
   $\therefore x = 1$.

10. If x is the required number, x - 13 is divisible by 18.
   $\therefore$ x - 13 is also divisible by 9.
   Dividing the remainder viz. 13 by 9, we get the remainder 4.
   Hence, the remainder when x is divided by 9 is 4.

11. 1 and 8 are the first two digits of the divisor and 3 and 6, those of the dividend.
Hence we try 200x187 = 37400.
Subtracting 374 twice from this, we get 36652.
We confirm that (36652 - 36520).
Also, 36652 - 36520 = 132. < 187.
Hence the least number that should be added is 132.

12. 467016 is divisible by 4 because it ends in 16, by 11 because (4+7+1) - (6+0+6) = 0, and by 3 because 4+6+7+0+1+6 = 24.
Hence 467016 is divisible by 3x4x11 i.e. by 132.

13. We have 100x193 = 19300.
$19300 - 17289 \approx 2000 \approx 193 \times 10$
Hence, 193x90 = 17370.
17370 - 17289 = 81
As 81 < 193, closest is 17370.

14. We have
$$\begin{aligned}(511)^2 &= (500 + 11)^2\\ &= 500^2 + 2 \times 500 \times 11 + 11^2\\ &= 250000 + 11000 + 121\\ &= 261121.\end{aligned}$$

15.
$$\begin{aligned}781 \times 819 &= (800 - 19)(800 + 19)\\ &= 800^2 - 19^2\\ &= 640000 - 361\\ &= 639639\end{aligned}$$

16. Subtracting 2 from both sides, we get, $x^2 - 2 + 1/x^2 = 18 - 2 = 16$.
$\therefore (x - 1/x)^2 = 16 \quad \therefore x - 1/x = \pm 4$

17. $(p - q)^2 = 5^2 = 25$
$\therefore p^2 - 2pq + q^2 = 25$
$\therefore p^2 - 2 \times 66 + q^2 = 25$, since p q = 66.
$\therefore p^2 + q^2 = 25 + 132 = 157.$

18. We have $(x + y + z)^3 = 9^3 = 729$.
$\therefore x^3 + y^3 + z^3 + 3xy(x+y) + 3yz(y+z) + 3xz(x+z) + 6xyz = 729$
adding and subtracting 3xyz and regrouping, we have,
$(x^3 + y^3 + z^3 - 3xyz) + 3xy(x+y+z) + 3yz(x+y+z) + 3xz(x+y+z) = 729.$
$\therefore (x^3 + y^3 + z^3 - 3xyz) + 3(xy + yz + xz)(x + y + z) = 729.$
$(x^3 + y^3 + z^3 - 3xyz) + 3 \times 11 \times 9 = 729.$
$x^3 + y^3 + z^3 - 3xyz = 432$

19. We have, X + Y + Z = (b + c - 2a) + (c + a - 2b) + (a + b - 2c) = 0
$\therefore X^3 + Y^3 + Z^3 - 3XYZ = (X+Y+Z)(X^2 + Y^2 + Z^2 - XY - YZ - ZX) = 0.$

20. Multiplying both sides by xyz, we get,
$x^3 + y^3 + z^3 = 3xyz. \Rightarrow (x^3 + y^3 + z^3 - 3xyz) = 0$
$\therefore (x + y + z)(x^2+y^2+z^2 - xy-yz-zx) = 0$ hence Answer cannot be determined. The result is True for all numbers where x=y=z.

21. Multiplying both sides of the first equation by abc, we get,
bc + ac + ab = 0 ....(1)
Also, $(a + b + c)^2 = 5^2$
$a^2 + b^2 + c^2 + 2(ab + bc + ac) = 25$ ....(2)
Substituting (1) in (2),
$a^2 + b^2 + c^2 = 25$
We have, $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc)$
L.H.S. = (5) (25 - 0)
$a^3 + b^3 + c^3 - 3abc = 125.$

22. The 24th term in the product will be (x - x)
But (x - x) = 0.
(x - a) (x - b) ... (x - z) = 0.

23. Since a = b = c = d , $(a - b)^2 + (b - c)^2 + (c - d)^2 + (d - a)^2 = 0$
Expanding each term, we get,
$2 (a^2 + b^2 + c^2 + d^2) - 2 (ab + bc + cd + da) = 0$
$a^2 + b^2 + c^2 + d^2 = ab + bc + cd + da$

24. L.H.S. = $a^2 + 2ab + b^2 + b^2 + 2bc + c^2 + c^2 + 2cd + d^2$
= $a^2 + 2b^2 + 2c^2 + d^2 + 2 (ab + bc + cd)$
= ab + 2bc + 2cd + ab + 2 (ab + bc + cd) (since $a^2 = ab$, $b^2 = bc$, $c^2 = cd$, $d^2 = ab$)
= 4 (ab + bc + cd)
= R.H.S.

25. (!) This number 3564328^{161} ends with 8 (i.e the unit digit is 8). Hence the power cycle of 8 has to be considered. The power cycle of 8 is 8, 4, 2, 6. This cycle gets repeated after every four powres. Hence 8^4, 8^8, 8^{12}………. 8^{160} will end with 6. The next power i.e. 8^{161} will end with 8. Hence the unit digit of 3564328^{161} will be 8.

(!!) The unit digits of the terms 61^{91}, 345^{201}, 176^{46} are 1,5 and 6 respectively. The power cycles of 1is 1,1,1,1 whereas that of 5 is 5,5,5,5 and that of 6 is 6,6,6,6. Hence whatever may be the powers of 61 it will end with 1 (unit's digit). Similarly any power of 345 and 176 will end in 5 and 6 respectively. So the unit digit of the expression $61^{91} + 345^{201} - 176^{46}$ will be 1+5-6 i.e. 0.

(!!!) The power cycle of 7 is 7,9,3,1 whereas that of 11 is 1. The 48th power of 7 will therefore have 1 as its unit digit whereas any power of 11 has to end with 1. Hence the unit digit of the expression $(7^{48} - 11^7)$ will be 1-1 i.e. 0. Hence this number will have a 0 as its unit digit and this number is completely divisible by 5 and will leave 0 as a remainder.

26. When a + b + c = 0, $a^3 + b^3 + c^3 = 3abc = -3ab(a + b)$ which is always even, irrespective of the nature of the numbers a and b.

27. $(9\sqrt{3} + 11\sqrt{2})^{1/3} = [(\sqrt{3})^3 + (\sqrt{2})^3 + 3.3.\sqrt{2} + 3.2.\sqrt{3}]^{1/3} = (\sqrt{2} + \sqrt{3})$.

28. $(20 + 14\sqrt{2})^{1/3} = (2^3 + (\sqrt{2})^3 + 3.4.\sqrt{2} + 3.2.2)^{1/3} = (2 + \sqrt{2})$
similarly $(20 - 14\sqrt{2})^{1/3} = (2 - \sqrt{2})$.
Sum of the two terms equals 4.

29. Let A/a = B/b =C/c =D/d = x.
$\therefore$ A = ax, B = bx, C = cx and D = dx,
$\therefore$ A + B + C + D = x (a + b + c + d).
Now $(Aa)^{1/2} + (Bb)^{1/2} + (Cc)^{1/2} + (Dd)^{1/2} = (a + b + c + d) \sqrt{x}$
$= [(A + B + C + D)/(a + b + c + d)]^{1/2}. (a + b + c + d)$
$= [(A + B + C + D).(a + b + c + d)]^{1/2}$.

30. $(8 + 7 + \varsigma +4) - (2+6+8+5) = \varsigma - 2$
= 0 or multiple of 11 .
$\therefore$Only digit at ς that makes the number divisible by 11 is 2.

31. Using the formula $(a^3 + b^3) = (a + b) (a^2 + b^2 - ab)$. Ans = 11.347 + 18.653 = 30

32. $(97 + 56\sqrt{3})^{1/4} = (49 + (4\sqrt{3})^2 + 2.7.4\sqrt{3})^{1/4} = (7 + 4\sqrt{3})^{1/2} = (4 + 3 + 2.2.\sqrt{3})^{1/2} = (2 + \sqrt{3})$.

33. The expression can be simplified to $(16)^{1.5} / (256)^{0.50} = (16)^{0.50} = \pm 4$.

34. Let a =26.21 and b = 14.79. The given sum is of the form:
$\frac{(a^2 - b^2)}{4.1(a - b)} = \frac{(a + b)}{4.1} = \frac{26.21 + 14.79}{4.1} = 10.$

35. To find the square root of a number upto four places of decimal, one should have four pairs after the decimal. So we add seven zeroes after 7

$$\begin{array}{r|l}
 & 0\ .\ 8\ 3\ 6\ 6 \\
\hline
8 & 0\ .\ 7\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\
+8 & \quad\ 6\ 4 \\
1\ 6\ 3 & \qquad 6\ 0\ 0 \\
+3 & \qquad 4\ 8\ 9 \\
1\ 6\ 6\ 6 & \qquad 1\ 1\ 1\ 0\ 0 \\
+6 & \qquad\ \ 9\ 9\ 9\ 6 \\
1\ 6\ 7\ 2\ 6 & \qquad\ \ 1\ 1\ 0\ 4\ 0\ 0 \\
 & \qquad\ \ 1\ 0\ 0\ 3\ 5\ 6 \\
 & \qquad\quad\ 1\ 0\ 0\ 4\ 4
\end{array}$$

36. $\frac{(0.023)^3 + (0.117)^3}{0.023 \text{x} 0.023 - 2.3 \text{x} 0.00117 + 1.17 \text{x} 0.117}$ Use $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= \frac{(0.023 + 0.117)(0.023^2 - 0.023 \text{x} 0.117 + 0.117^2)}{(0.023^2 - 0.023 \text{x} 0.117 + 0.117^2)} = (0.023 + 0.117) = 0.14.$

37. $\frac{0.07 \text{x} 0.07 - 0.07 \text{x} 0.03 + 0.03 \text{x} 0.03}{0.07 \text{x} 0.07 \text{x} 0.07 + 0.03 \text{x} 0.03 \text{x} 0.03} = \frac{(0.07^2 - 0.07 \text{x} 0.03 + 0.03^2)}{(0.07 + 0.03)(0.07^2 - 0.07 \text{x} 0.03 + 0.03^2)}$

$= \frac{1}{(0.07 + 0.03)} = \frac{1}{0.1} = 10.$

38. Use $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a^2 - b^2) = (a + b)(a - b)$

39. Use $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a^2 - b^2) = (a + b)(a - b)$

40. Use $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$(a^2 - b^2) = (a + b)(a - b)$

41. Use $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

42. Use $(a^2 - b^2) = (a + b)(a - b)$

43. The given expression can be simplified as (0.008 + 0.027)/(0.04 – 0.06 + 0.09). This expression is in the form $(a^3 + b^3)/(a^2 - ab + b^2) = (a + b)$. Therefore, the required answer is (0.2 + 0.3) = 0.5.

44. $(x^n - y^n)$ is always divisible by (x – y). We can write $(9^6 + 1)$ as $(9^6 - 1 + 1 + 1)$ i.e. $[(9^6 - 1^6) + 2]$. Now $(9^6 - 1^6)$ is completely divisible by 8. So when $[(9^6 - 1^6) + 2]$ is divided by 8 remainder will be 2 only.

45. 100! = 100 x 99 x 98 x 97 x 96 ....3 x 2 x 1.
Number of multiples of 3 = 99/3 = 33, we are taking 99 instead of 100 because 99 is the highest multiple of 3 less than 100. Each of these 33 numbers will give 1 factor of 3.

Number of multiples of square of 3(i.e. 9) = 99/9 = 11, again we are considering 99 because 99 is the highest multiple of 9 less than 100. Each of these 11 numbers will provide 2 factors of 3, we have already considered 1 factor from each above, so we will get 11 additional factors of 3. Number of multiples of cubes of 3(i.e. 27) = 81/27 = 3, each of these three numbers will give us 3 factors of 3. We have already considered 2 factors of each of these numbers so we will get 3 additional factors.
Similarly 81 = 3^4 and it contains 4 factors of 3, we have already considered 3 factors this means we will get only 1 additional factor of 3.
Thus total factors of 3 obtained = (33 + 11 + 3 + 1) = 48. Thus 3^{48} is the highest power of 3 that will exactly divide 100!.

46. This question can be easily solved using the concept of power cycle.
$7^1 = 7$, digit at unit's place is 7.
$7^2 = 49$, digit at unit's place is 9.
$7^3 = 343$, digit at unit's place is 3.
$7^4 = 2401$, digit at unit's place is 1.
Now after this depending on the power of 7 the digits at unit's place will be repeated in a cycle i.e. for 7^5 digit at unit's place will be again 7, for 7^6 digit at unit's place will be 9 and so on.
In order get 7^{71} one has take 17 complete cycles(68/4, as 68 is the highest multiple of 4 less than 71) and then 3 more places i.e. unit's digit will be same as 7^3. Hence the answer is 3.

47. $2^{300} = (2^3)^{100} = 8^{100}$
$3^{200} = (3^2)^{100} = 9^{100}$
$5^{150} = (5\sqrt{5})^{100}$
Now in all the 3 cases the powers are same so the number with highest base will be the greatest, hence the answer is $(5\sqrt{5})^{100}$ as $\sqrt{5} > 2$ so $5\sqrt{5} > 9$.

48. If a number has 'n' terminal zeros, then it is divisible by 10^n . So we have to find how many factors of 10 are contained in 100!. But since 10 = 5 x 2, we have to find how many factors of 5 & 2 are there. Since 2 is smaller than 5, for any factor of 5 there will be enough factors of 2 to make a factor of 10. Thus we need to count only factors of 5.
Since 100 = 20 x 5, there are 20 multiples of 5 in the product 1 x 2 x 3 - - - - - -99 x 100. But there are more factors of 5, since the numbers 25, 50, 75 and 100 (square of 5 and its multiples) will contain 2 factors of 5, to make 4 'extras'. There are therefore 24 factors of 5, so 24 factors of 10, so 24 terminal zeros in the product 100!.

49. $(x^n - y^n)$ is always divisible by $(x - y)$. We can write 7^{100} as $[7^{100} - 1^{100} + 1]$. Now $(7^{100} - 1^{100})$ is completely divisible by 6. So when $[7^{100} - 1^{100} + 1]$ divided by remainder will be 1 only.

50. $2^5 = 32$, digit at the unit's place is 2.
$3^5 = 243$, digit at the unit's place is 3.
$4^5 = 1024$, digit at the unit's place is 4.
$5^5 = 3125$, digit at the unit's place is 5.and so on i.e. if x is a single digit natural number then the number at the unit's place of x^5 will be x itself.