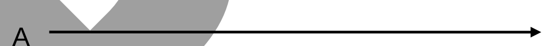
Concepts 1

Geometric Fundamentals

A few basic facts are assumed in geometry. These facts are called **Postulates** or **Axioms.** Axioms are not proved, their truth is taken for granted. Following are the axioms :

- Space contains at least two distinct points.
- A line is the shortest distance between two points. Every line is a set of points and contains at least two distinct points.
- Given any two distinct points in a plane, there exists one and only one line containing them.
- No line contains all the points of the space.
- A plane is a set of points and contains at least three non-collinear points.
- If there are three non-collinear points then there is one and only one plane that contains all of them.
- No plane contains all the points of space.
- If two distinct points of a line lie in a plane, then every point of the line lies in that plane.
- If two distinct planes intersect then their intersection is a line.

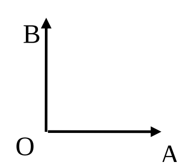
A **Ray** extends infinitely in one direction from any given point. This is exhibited by an arrow. The starting point, say A, of the ray is called the **initial point**.



An angle is a figure formed by two rays with a common initial point, say O. This point is called the **vertex.**

Types of Angles

A right angle is an angle of 90^o. e.g. Angle AOB

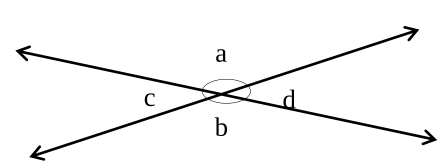


An angle less than 90^o is called **acute**, an angle greater than 90^o but less than 180^o is called **obtuse**, an angle of 180^o is called a **straight angle**, an angle greater than 180^o but less than 360^o is called a **reflex angle**.

Two angles whose sum is 180^o are called **supplementary** angles, each one is a supplement of the other. Two angles whose sum is 90^o are called **complementary** angles, each one is a complement of the other. Two adjacent angles whose sum is 180^o are the angles of a linear pair.

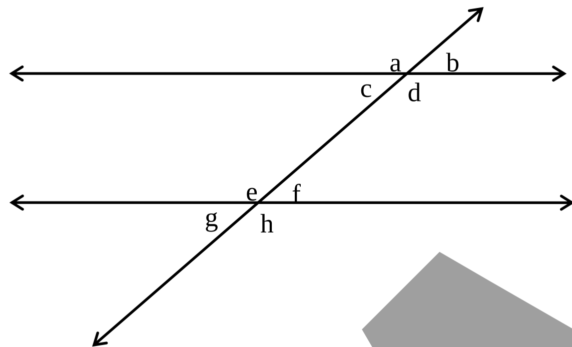
Angles and Intersecting lines : When two lines intersect, two pairs of vertically opposite angles are formed. Vertically opposite angles are equal. Thus, $\angle c$ & $\angle d$ are equal. $\angle a$ & $\angle b$ are equal. Also, sum of all the angles at a point = 360^o.

$$\angle a + \angle b + \angle c + \angle d = 360^o$$



Angles and Parallel Lines : If a transversal (cutting line) cuts two parallel lines, then the

- Corresponding angles are congruent i.e. a = e, b = f, d = h, c = g
- Alternate angles are congruent, i.e. c = f, d = e.
- Interior angles on the same side of the transversal are supplementary; c + e = d + f = 180^o



Two lines are parallel to each other :

- If alternate angles made by a transversal are congruent.
- If the corresponding angles made by a transversal are congruent.
- If the interior angles on the same side of the transversal are supplementary.
- If they are parallel to a third line.
- If they are perpendicular to a third line.
- If they are the opposite sides of a parallelogram, rectangle etc.
- If one of them is a side of a triangle and the other joins the midpoints of the remaining two sides.
- If one of them is a side of a triangle and the other divides the other two sides of the triangle proportionally.

Two lines are perpendicular to each other :

- If the adjacent angles formed by them are equal and supplementary.
- If one of them is the internal bisector and the other is an external bisector of an angle.
- If they are parallel to the arms of a right triangle making the right angle.
- If they are the adjacent sides of a rectangle or a square.
- If they are the diagonals of a rhombus.
- If the sum of the squares on them is equal to the square on the line joining their free hands.
- If one of them is a tangent to a circle and the other is the radius of the circle through the point of contact.

Two angles are congruent :

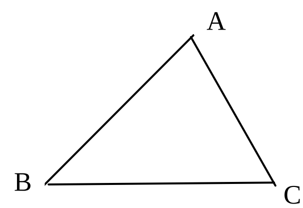
- If they are the complements of congruent angles.
- If they are the supplements of congruent angles.
- If they are vertically opposite angles.
- If they are alternate angles formed by a transversal and parallel lines.
- If they are corresponding angles formed by a transversal and parallel lines.
- If their arms are parallel to each other in the same sense.
- If their arms are perpendicular to each other.
- If they are the corresponding angles of two congruent triangles.
- If they are the opposite angles of a parallelogram.
- If they are the angles of an equilateral triangle.
- If they are the opposite angles of the congruent sides of an isosceles triangle.
- If they are the angles of a regular polygon.
- If they are subtended by congruent arcs in the same circle or in congruent circles at the centre(s).
- If they are subtended by congruent arcs in the same circle or in congruent circles at their circumferences.
- If they are in the same segment of a circle.
- If they are such that one of them is the exterior angle and the other is the interior opposite angle of a cyclic quadrilateral.
- If one of them lies between a tangent and a chord through the point of contact and the other is in the alternate segment, in a circle.

Two segments are congruent :

- If they are the corresponding sides of two congruent triangles.
- If they are the sides opposite to the congruent angles of a triangle.
- If they are the sides of an equilateral triangle.
- If they are the opposite sides of a parallelogram.
- If they are the sides of a regular polygon.
- If they are the intercepts on a transversal made by parallel lines, which make congruent intercepts on another transversal.
- If they are the radii of the same circle or congruent circles.
- If they are chords equidistant from the centre of the circle.
- If they are the chords of congruent arcs in the same circle or in congruent circles.
- If they are tangents to a circle from an external point.
- If they are perpendiculars from a point on the bisector of an angle to its arms.

Concepts 2

A Triangle is a closed figure formed by three line segments. In the figure below, ABC is a triangle having sides AB, BC and CA. A, B, C are the vertices of the triangle.
A triangle is said to have six elements. Three sides AB, BC and CA and three angles ∠A, ∠B & ∠C. A triangle may also be defined as polygon of three sides.

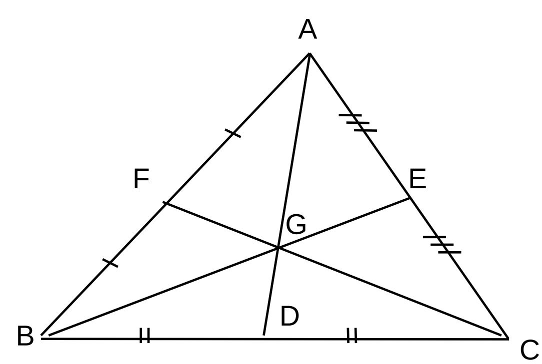


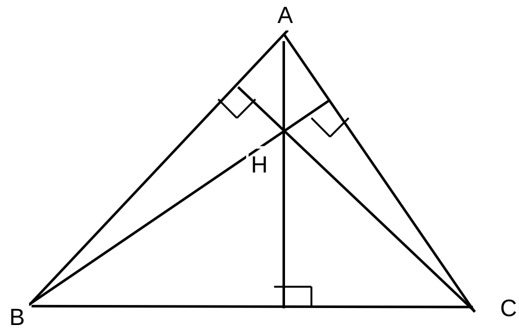
- If any two sides of a triangle are equal it is called an **isosceles** triangle.
- When all the three sides are equal it is called an **equilateral** triangle.
- A triangle with no two sides equal is called a **scalene** triangle.
- If one of the angles of a triangle is 90^o it is called a **right-angled** triangle.
- If one of the angles of a triangle is more than 90^o it is called an **obtuse-angled** triangle.
- If all the angles of the triangle are less than 90^o it is called an **acute-angled** triangle.

Basic Properties of a triangle :

- Sum of the three angles is 180^o.
- When one side is extended in any direction an angle is formed with another side. This angle is called the **exterior angle**. There are six exterior angles of a triangle.
- The sum of an angle of a triangle called interior angle and the exterior angle adjacent to it is 180^o.
- An exterior angle is equal to the sum of the two interior angles not adjacent to it.
- Sum of any two sides is always greater than the third side.
- Difference of any two sides is always less than the third side.
- Side opposite to the largest angle will be the greatest side.
- Side opposite to the smallest angle will be the shortest side.
- Two triangles will have equal area if they have the same base and they lie between the same parallels.
- In any triangle there can be only one right angle or obtuse angle. i.e. a triangle must have at least two acute angles.

The segment joining a vertex and the midpoint of the opposite side is called the **median** of a triangle. There are three medians and they meet in a single point called the **centroid** of the triangle, & denoted by **G.** The centroid divides each median in the ratio 2 : 1. In the given figure, AG: GD = BG: GE = CG: GF = 2 : 1.





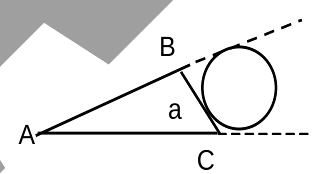
The line drawn from any vertex, perpendicular to the opposite side is called the **altitude (or height).** The three altitudes of a triangle meet in a single point called the **orthocentre.** It is denoted by **H.** The angle made by any side at the orthocentre is equal to (180 - vertical angle)

Incentre : Point of intersection of angle bisectors of the triangle is known as the **Incentre** of the triangle and it is denoted by **I**. Circle drawn with this point as the centre and touching all the three sides of the triangle is known as **Incircle**. Radius of this circle is known as **Inradius**, denoted by r and

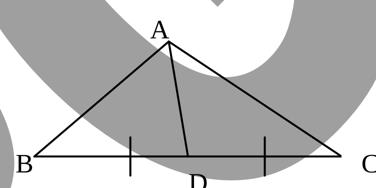
r = Δ/ s, where Δ = Area of the triangle and s = semi-perimeter.

<u>Circumcentre</u> : is the point of intersection of perpendicular side bisectors of the triangle. Circle drawn with this as the centre and passing through the vertices is **Circumcircle**, and radius of this circle is **Circumradius** (R) and R = abc/ 4Δ = a / (2 Sin A).

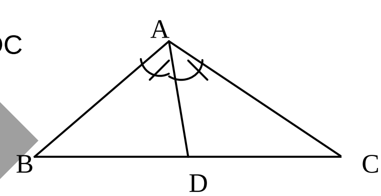
<u>Ex-circle</u> : If ABC is any triangle then the circle touching to side BC and AB, AC produced is known as the **Ex-circle** opposite to A. Its radius is r_a = Δ / (s - a)



Apollonius' theorem: If AD is the Median of the given triangle ABC , then , $AB^2 + AC^2 = 2(AD^2 + BD^2)$



Angle Bisector theorem : The angle bisector of a triangle divides the opposite side in the ratio of its adjacent arms .
If AD bisects ∠A, then AB/AC = BD/DC



Area of a triangle :
Area of a triangle = 1/2 x Base x height

Hero's Formula : If a, b, c are the lengths of the three sides of a triangle, then

area = $\sqrt{s(s-a)(s-b)(s-c)}$ where s = (a + b + c) /2

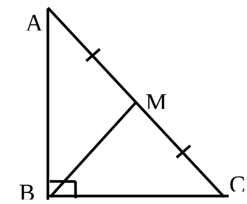
Properties of Different types of triangles **:**

Equilateral triangle :

- All sides are equal, all angles are equal. Each angle = 60^o
- Height = (√3/2)side; Area = (√3/4)$side^2$ = h^2 / √3

- Inradius = height/3 = a / 2√3; circumradius = 2(height)/3 = a / √3
- Perimeter = 3 x side
- Of all the triangles of any given perimeter, equilateral triangle has the maximum area.
- Of all the triangles that can be inscribed in a circle equilateral triangle has the maximum area.

Right angled triangle :



- In a right angled triangle, one angle is a right angle, the side opposite to the right angle is called **hypotenuse**. The other two acute angles are complementary.
- Square of the hypotenuse = Sum of squares of the other two sides. (**Pythagoras Theorem**) i.e. $AC^2 = AB^2 + BC^2$
- Pythagoras theorem has a large number of applications. Hence, it is better to remember the Pythagorean triplets.

	+2	+2	+2	+2	+2	+2
3	**5**	**7**	**9**	**11**	**13**	**15**
	+8	+12	+16	+20	+24	+28
4	**12**	**24**	**40**	**60**	**84**	**112**
	+8	+12	+16	+20	+24	+28
5	**13**	**25**	**41**	**61**	**85**	**113**

	+4	+4	+4	+4	+2
4	**8**	**12**	**16**	**20**	**24**
	+12	+20	+28	+36	+44
3	**15**	**35**	**63**	**99**	**143**
	+12	+20	+28	+36	+44
5	**17**	**37**	**65**	**101**	**145**

The largest is the hypotenuse and same multiples of these triplets are also Pythagorean triplets. e.g. **3, 4, 5:** 6, 8, 10 ; 9, 12, 15 ; 12, 16, 20, . . .

- The median to the hypotenuse = (1/2) hypotenuse. This median is also the circumradius of the triangle.
- Area = (½)product of perpendicular sides

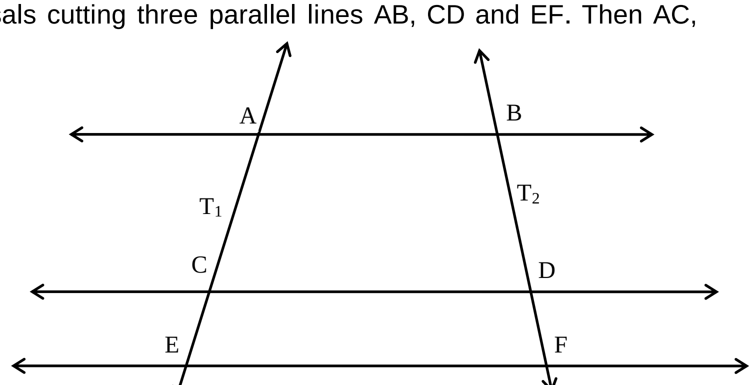
Isosceles right triangle :

- In an isosceles right triangle each acute angle is 45^o.
- Hypotenuse = (√2) x (one of the perpendicular arms)

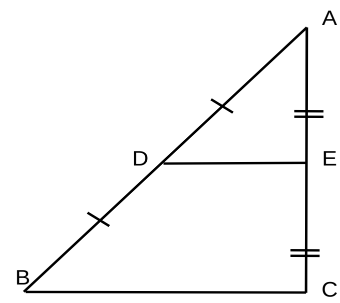
30-60-90 triangle :

- the side opposite to 30^o = (1/2) hypotenuse
- the side opposite to 60^o = (√3/2) hypotenuse

Midpoint Theorem : Intercepts, made by two transversals on 3 or more parallel lines are proportional. Lines t_1 and t_2 are transversals cutting three parallel lines AB, CD and EF. Then AC, CE, BD, DF are the intercepts.
AC/CE = BD/DF;
If AC = CE then BD = DF



In a triangle, the segment joining the midpoints of two sides is parallel to the third side and is equal to half the third side.



DE || BC and DE = $^{1}/_{2}$ BC **(midpoint theorem)**

Basic Proportionality theorem (BPT): A line parallel to one side of a triangle, divides the other two sides proportionally.

Congruency of triangles :

Two triangles are congruent:

- If two sides and the included angle of one triangle are respectively congruent to two sides and the included angle of the other triangle. (**S - A - S**)
- If two angles and a side of one triangle are respectively congruent to two angles and the corresponding side of the other. (**A - A - S**)
- If three sides of one triangle are respectively congruent to the three sides of the other.(**S-S-S)**
- If one side and hypotenuse of a right angled triangle are respectively congruent to the side and the hypotenuse of the other right angled triangle **(R.H.S.)**.
- If two angles and the side included between them of one triangle are respectively congruent to two angles and the corresponding side of the other triangle (**A -S- A**)

Similarity of triangles :
Two triangles are similar if they are alike in shape only. The corresponding angles are congruent, but the corresponding sides are only proportional. All congruent triangles are similar but all similar triangles are not necessarily congruent.

Two triangles are similar if :

- Three angles of one triangle are respectively equal to the three angles of the other triangle. (A - A - A)
- Two angles of one triangle are respectively equal to the two angles of the other triangle. (A - A)
- Two sides of one triangle are proportional to two sides of the other and the included angles are equal.

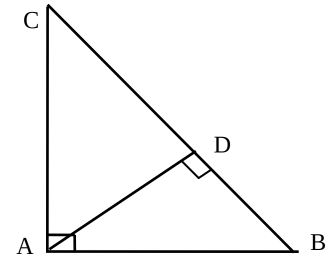
Properties of similar triangles :

If two triangles are similar,
Ratio of sides = ratio of heights = ratio of medians = ratio of angle bisectors = ratio of inradii = ratio of circumradii.

Ratio of areas = $b_1h_1/b_2h_2 = (s_1)^2/(s_2)^2$, where b_1 and h_1 are the base and height of Δ_1, and b_2 and h_2 are the base and height of Δ_2; s_1 and s_2 are the corresponding sides of Δ_1 and Δ_2 respectively.

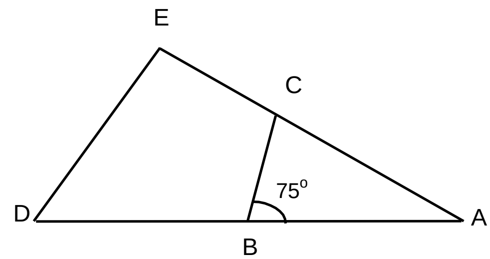
Similarity in a right angled triangle :

In a right triangle, the altitude to the hypotenuse separates the triangle into two triangles which are similar to each other and to the original triangle.
In ΔABC, $\angle A = 90^o$ $AD \perp BC$.
$\Delta ABC \sim \Delta DBA \therefore BA^2 = BC \times BD$
$\Delta ACD \sim \Delta BCA \therefore CA^2 = CB \times CD$
$\Delta ABD \sim \Delta CAD \therefore DA^2 = DB \times DC$



Solved Examples

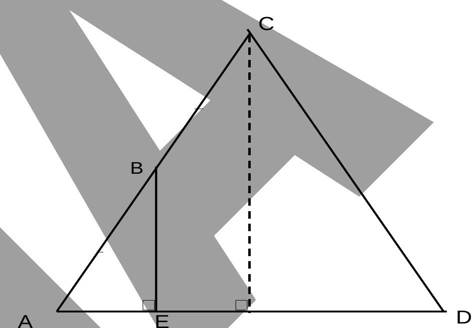
1. Line BC divides triangle ADE into sections, one of which is an isosceles triangle such that AB = AC. Angle B, one of the base angles, is equal to 75°. What is the sum of the measures of angles D and E?



(a) 100° (b) 125° (*c) 150° (d) 175°

The two base angles of an isosceles triangle are equal. Angles B and C both, therefore, equal 75^o and their sum is 150^o. Since a triangle has 180^o, angle A must be 30^o. Again, since a triangle has 180^o, the sum of angles A, E and D must be 180^o. But since A is known to be $30^{o,}$ the sum of angles D and E must be equal to 150^o.

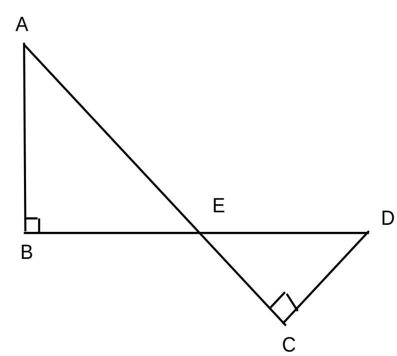
2. In the diagram, AC = CD = DA, AB = BC, and BE is perpendicular to DA. If the length of AE is 1, what is the area of triangle ACD?



(a) √3 (b) 2√3
(c) 3√3 (*d)4√3

Sol : In an equilateral triangle, each angle is equal to 60^o. The perpendicular dropped from B forms a 30-60-90 triangle with AB and AE. In a 30-60-90 triangle, the side opposite the 30^o angle is half the hypotenuse, while the side opposite the 60^o angle is $^{\sqrt{3}}/_2$ times the hypotenuse. As AE is 1, AB the hypotenuse must be 2. As the side of the triangle is twice AB, the side must be 4. The altitude from C is 2√3. Half the base of the equilateral triangle is 2 since it is opposite a 30^o angle. The area of a triangle is equal to half the product of base and altitude, therefore the area of the triangle is 4√3.

3. In the figure, if AB = BE = ED = 1, then AC = ?

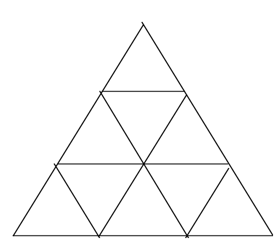


(a) 2 (*b) $^3/_2\ \sqrt{2}$

(c) 1 + √2 (d) √3

Sol : Since AB = BE, ABE is an isosceles triangle and angle DEC = 45^o. ∴ EDC is also an isosceles right angled triangle. By the Theorem of Pythagoras, AE = √2, EC = 1/√2. ∴ AC = $^3/_2\ \sqrt{2}$.

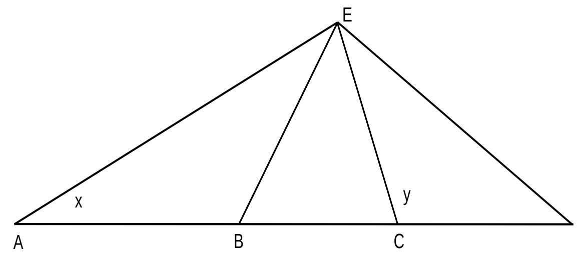
4. If more small triangles were added to the figure so as to double each of the sides of the outside triangle, how many small triangles would there be in the figure?



(a) 25 (b) 18
(*c) 36 (d) 40

Sol : To double the sides, we must draw three more rows of small triangles. Each row contains two more triangles. Each row contains two more triangles than the previous row. Altogether there are 1 + 3 + 5 + 7 + 9 + 11 = 36 small triangles.

5. In the diagram, AB = BE and BC = EC. If points A,B,C, and D are collinear, find y.

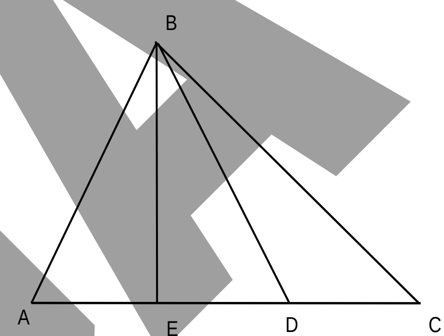


(a) x (b) 1/2x

(c) 1/3x (*d) 4x

Sol : AB = BE and BC = EC . In triangle ABE, angle BAE = angle AEB = x. In triangle BEC, angle BEC = angle CBE = y. CBE is the exterior angle of triangle ABE, angle CBE = angle BAE + angle AEB = 2x. Similarly, angle DCE is the exterior angle of triangle CBE. Therefore, y = angle DCE = angle BEC + angle CBE = 4x.

6. In triangle ABC, AE = DE =DC. If the area of triangle BEC is 48, what is the area of triangle ABC?



(a) 64 (*b) 72
(c) 96 (d) 80

Sol : Triangles ABC and BEC share altitudes to side AC and EC = $^2/_3$ AC; Area of triangle BEC = ($^2/_3$)(Area of triangle ABC); Area of triangle ABC = 72

Polygons :

A polygon with:

3 sides is called a **triangle**

4 sides is called a **quadrilateral (square, rectangle, rhombus, parallelogram, trapezium)**

5 sides is called a **pentagon**

6 sides is called a **hexagon**

7 sides is called a **heptagon**

8 sides is called an **octagon**

9 sides is called a **nonagon**

10 sides is called a **decagon**

infinite sides is called a **circle.**

A polygon with all sides and all angles equal is called a **regular polygon.** A regular polygon can be inscribed in a circle.

Cyclic Polygon : If a circle can be drawn passing through all the vertices of a polygon then it is known as a cyclic polygon.

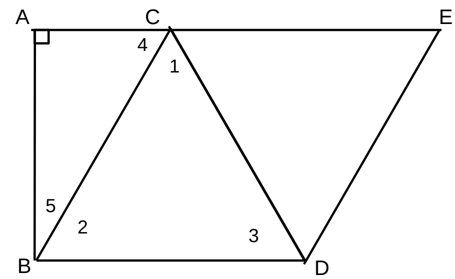
Properties of regular polygons :

If the number of sides of the polygon is n,

- sum of all interior angles = (2n - 4) x 90^o

- an interior angle + an exterior angle = 180^o
- Area = $^1/_2$ perimeter x perpendicular from centre to any side.

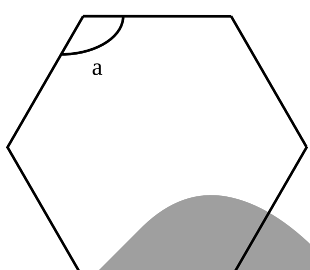
7. In the diagram, Angles 1,2,and 3 are equal. Triangle BCD is congruent to triangle CDE. Angle CAB = 90°. A,C, and E are collinear. BC = 5. What is the perimeter of quadrilateral ABDE?

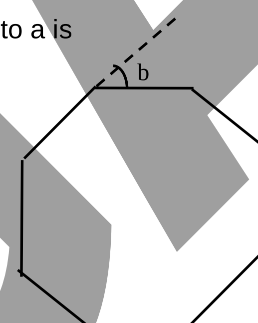


(a) 15 (b) $22\ ^1/_2$
(*c) $(35 + 5\sqrt{3})/2$ (d) $(10 + 5\sqrt{2}) / 2$

Sol : Since angles 1, 2 and 3 are equal triangle BCD is equilateral and all its sides are equal. BC = 5, then BD = CD = 5. Since triangle BCD and triangle CDE are congruent, DE = CE = 5. Each angle of an equilateral triangle equals 60^o. $\therefore$ angle 1 + angle DCE = 120^o. Angle 4 = 180 - 120 = 60^o. Angle 5 = 180 - (90 + 60) = 30^o. Since the side opposite the 30^o angle is equal to half the hypotenuse (BC), AC = 2.5. By the theorem of Pythagoras, AB = $(5\sqrt{3})$ / 2. The perimeter of ABDE = AB + BD + DE + CE + AC.

8. The figure shows two regular polygons. The ratio of b to a is





(a) 9/8 (b) 8/9 (*c) 3/8 (d) 2/7

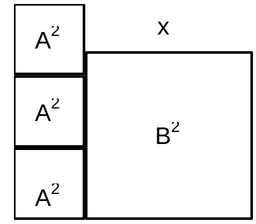
Each interior angle of an n-sided regular polygon measures $^{(n-2)}/_n$ 180 Therefore for the first figure, a hexagon, angle a is $^{(6-2)}/_6$ 180 = 120^o.Each exterior angle is found by taking 360/n. This means that angle b, an exterior angle of an octagon, is 360/8 = 45^o. $\therefore$b/a = 3/8

Quadrilaterals :
In a quadrilateral sum of all four angles = 360^o
Area of a quadrilateral = $(^1/_2)$ (one diagonal)x(sum of perpendiculars from opposite vertices).
A quadrilateral inscribed in a circle is a cyclic quadrilateral. The opposite angles in this case are supplementary. The exterior angle = the remote interior angle.

Square :

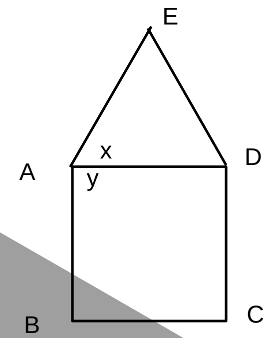
- A square has all sides equal.
- All angles are right angles. The diagonals are equal and bisect each other at right angles.
- Perimeter of a square = 4a where a is the length of a side
- Area = a^2 diagonal = $\sqrt{2}$ a
- When a square is inscribed in a circle, the diagonal = the diameter of the circle
- When a circle is inscribed in a square, side of the square = diameter of the circle.

9. In the figure, the area of each little square is A^2 and the area of the big square is B^2. Express x in terms of A and B.
(a) A + B (b) 3A/B (*c) 3A - B (d) $3A^2 - B^2$



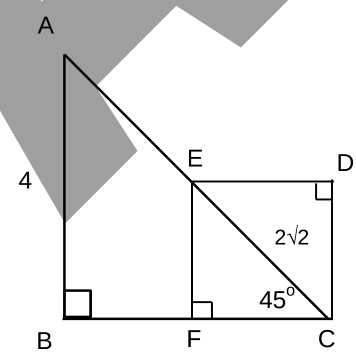
Sol : The area of the little squares is A^2. $\therefore$ the each side is A units long. Similarly, each side of the larger square is B units long. The height of the entire stack of small squares is 3A. The height of the large square is B. x is the difference between these two heights, 3A - B

10. The figure is composed of an equilateral triangle and a square.
angle (x + y) / Angle B = ?
(*a) 5/3 (b) 9/5
(c) 3/2 (d) 7/4



Sol : Angle A is composed of one angle of the square plus one angle of the equilateral triangle. $\therefore$ angle A = 150^o.
Angle B = 90^o. 150/90 = 5/3

11. What is the ratio of the area of square CDEF to the area of isosceles triangle ABC? Given AB = 4 & EC = $2\sqrt{2}$
(*a) 1:2 (b) 1:1
(c) 2:3 (d) 3:4



Sol : Δ ABC is isosceles right angled triangle. BC = AB = 4. Area of triangle ABC=(0.5)(4)(4)=8.
Using the theorem of Pythagoras, EF = 2.

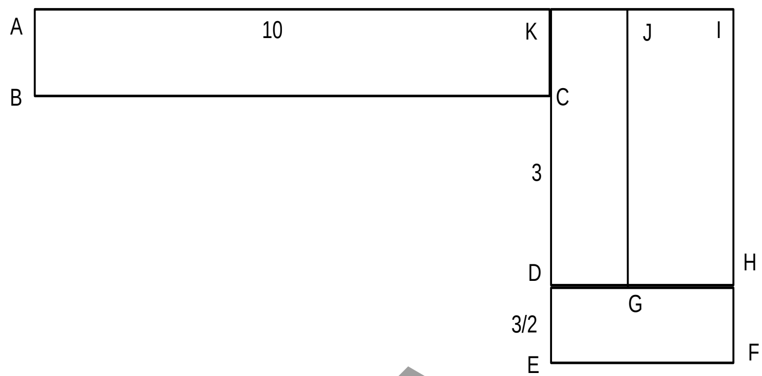
Rectangle :

- Opposite sides are equal; all angles are right angles.
- Diagonals are equal and bisect each other.
- Perimeter = 2(l + b) where l = length & b = breadth
- Area = l x b
- Diagonal = $\sqrt{(a^2 + b^2)}$
- Of all the rectangles of given area or perimeter the square will have the maximum area.
- In a rectangular box of length l, breadth b and height h, the length of the longest rod that can be kept is $\sqrt{(l^2 + b^2 + h^2)}$
- When the rectangle is inscribed in a circle , it will have the maximum area when it is a square.

12. Prasad has a sheet of paper 8 inches long and 3 inches wide,. How many strips of paper 6 inches long and 1 inch wide can he cut from this sheet?
(*a) 3 (b) 4 (c) 6 (d) 24

There would be 4 strips of 6 square inches a piece, one of the strips would measure 2 inches by 3 inches. There are only three strips measuring 6 inches by 1 inch.

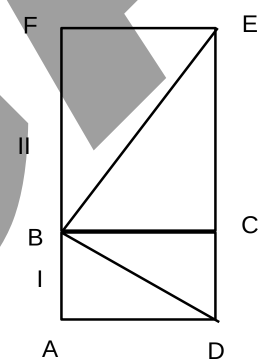
13. In the diagram below, AK is 10 units long, CD is 3 units long, DE is 3/2 units long and ABCK has an area of 20, KJGD has an area of 5, and DHFE has an area of 6. Find the area of IJGH.



(a) 10 (b) 20 (c) 12 (*d) 15

Sol : Area = length x width. In ABCK, 10 x CK = 20 $\therefore$ CK = 2, DK = 5, JG = 5.
In KJGD, 5 x DG = 5, $\therefore$ DG = 1.
In DHFE, 1.5 x DH = 6, $\therefore$ DH = 4
GH = DH - DG = 4 - 1 = 3.
Area of IJGH = JG x GH = 5 x 3 = 15

14. In the fig, the ratio of the area of triangle I to the area of triangle II is 1:3. What is the ratio of the area of rectangle ABCD to the area of the rectangle BCEF?

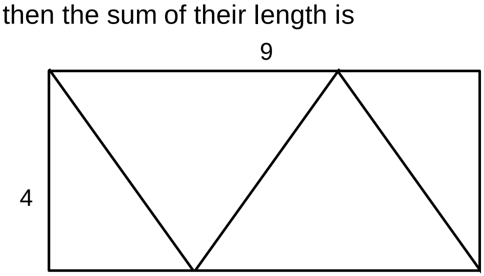


(*a) 1:3 (b) 2:3

(c) 1:2 (d) 1:9

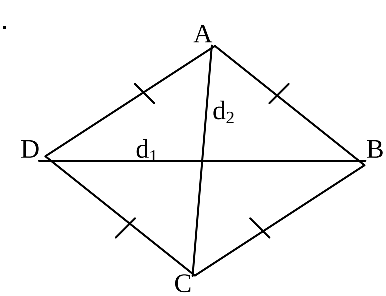
Sol : The area of triangle I is half the area of rectangle ABCD. Similarly, area of triangle II is half the area of rectangle BCEF. (area triangle I)/(area triangle II) = area ABCD/area BCEF = 1/3

15. If the three segments inside the rectangle are equal, then the sum of their length is
(a) 13 (b) 12 (c) 18 (*d) 15



Sol : After we draw the altitudes of every triangle, all the triangles are congruent. $\therefore$ each segment inside the rectangle cuts off a length 3 from the long side of the rectangle. The length squared of the inside segment is $3^2 + 4^2 = 5^2$. Thus each inside segment is 5 and their sum is 15.

Rhombus :

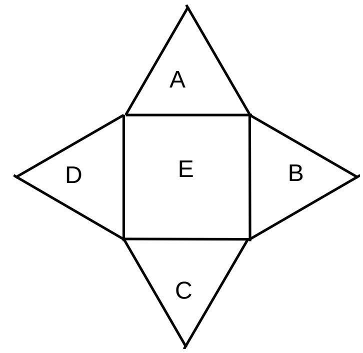


- A rhombus has all the sides equal and its opposite sides are parallel.
- Opposite angles are equal
- The diagonals bisect each other at right angles, but are not equal
- Area = $^1/_2\ d_1\ d_2$ where d_1 & d_2 are the two diagonals.
- $\text{Side}^2 = (d_1/2)^2 + (d_2/2)^2$

16. In the adjacent figure, triangles A, B, C, D are all congruent equilateral triangles, what can be said about quadrilateral E?

(a) E is a rectangle (b) E is a square

(*c) E is rhombus (d) None of the above is true

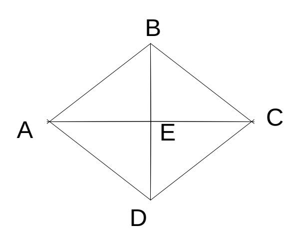


Sol : The sides of E are the bases of congruent triangles, they must all be equal. E is ∴ a rhombus. Though square may be correct in some cases, it can not be determined definitely.

Parallelogram :

- Opposite sides are parallel and equal.
- Opposite angles are equal.
- Diagonals bisect each other.
- Sum of any two adjacent angles = 180^o
- Bisectors of the four angles enclose a rectangle.
- Each diagonal divides the parallelogram into two triangles of equal area.
- A parallelogram inscribed in a circle is always a rectangle.
- A parallelogram circumscribed about a circle is always a rhombus.
- Straight lines joining the midpoints of adjacent sides of any quadrilateral form a parallelogram.
- Area = Base x height

17. In the figure parallelogram ABCD is composed of four congruent triangles. If BE is 3 and CE is 4 what is the perimeter of the entire figure?



(a) 24 (*b) 20

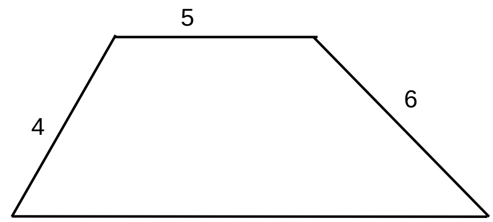
(c) 16 (d) 28

Sol : Since the four triangles are congruent, angles AEB, BEC, CED and DEA must be all equal. Their sum is 360^o. ∴ each angle must be 90^o. In triangle BEC, $(BE)^2 + (CE)^2 = (BC)^2$ ∴ BC = 5. The four sides of the figure must be equal. ∴ the perimeter = 4 x 5 = 20.

Trapezium:

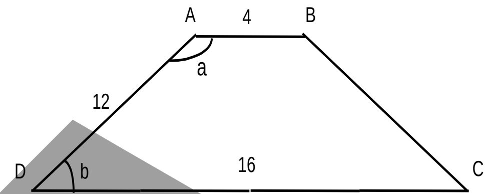
- A trapezium has only one pair of opposite sides parallel.
- area = $^1/_2$ (sum of parallel sides) x (height)
- The median is half the sum of parallel sides.
- A trapezium inscribed in a circle is a isosceles trapezium. In an isosceles trapezium the oblique sides are equal. Angles made by each parallel side with the oblique side are equal.

18. The perimeter of the figure is
(a) a whole number (*b) less than 30 (c) greater than 40 (d) 22



Sol : The remaining side must be less than 4+5+ 6, since the shortest distance between two points is a straight line. The perimeter is thus less than 30.

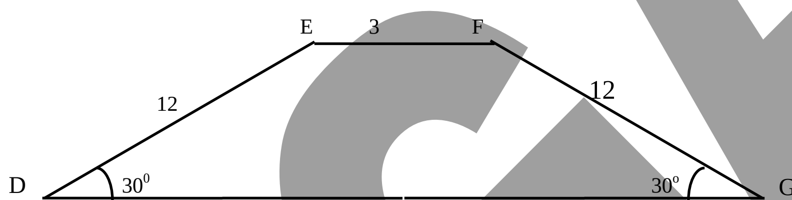
19. If trapezoid ABCD is with AB = 4, DC=16 and altitude of the trapezoid is 6, then a - b = ?



(a) 100° (b) 90°
(c) 45° (*d) 120°

Sol : Draw altitude AE and BF. Triangle DAE is 30-60-90^o so b = 30^o. ∴ a = 180 - 30 = 150^o. ∴ a - b = 150 - 30 = 120^o

20. What is the area of isosceles trapezoid DEFG?



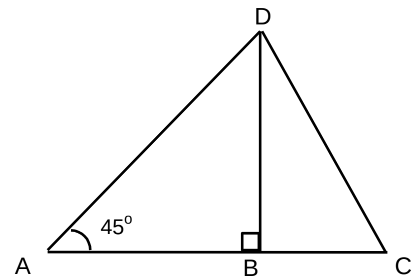
(a) 9 (b) 9+36√3 (c) 36 (*d) 18 + 36√3

Sol : Divide the trapezium into two triangles and a rectangle by dropping perpendiculars from E & F. Now the area of the trapezium will be sum of areas of the two triangles and the rectangle.

Diagonals	Parallelogram	Rectangle	Rhombus	Square
Bisect each other	✓	✓	✓	✓
Are equal		✓		✓
Bisect vertex ∠s			✓	✓
Are perpendicular			✓	✓
Form 4 equal Δs			✓	✓
Form 4 congruent Δs			✓	✓

Exercises

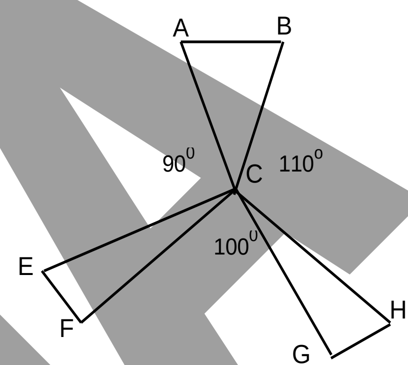
1. The area of the triangle is 30. BD is perpendicular to AC. AC is 10 units long and angle A is 45°. Find the length of DC.



(a) $\sqrt{26}$ (b) $2\sqrt{13}$

(c) 6 (d) $4 + \sqrt{2}$

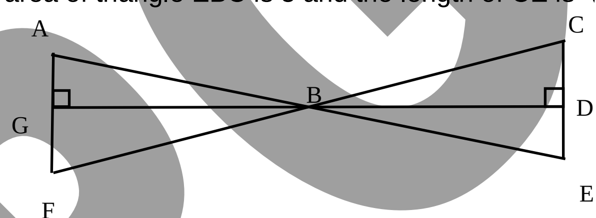
2. The three triangle shown above are isosceles and congruent. What is the measure of angle CAB?



(a) 80 (b) 66

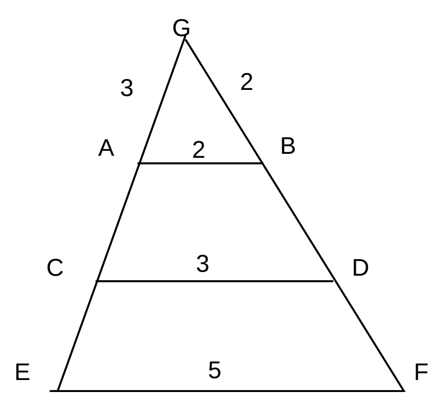
(c) 48 (d) 75

3. In the diagram, triangles ABF and EBC are congruent. The area of triangle EBC is 5 and the length of CE is $\sqrt{2}$. Find the length of GD.



(a) 4 (b) $5\sqrt{2}$ (c) $2\sqrt{3}$ (d) $10\sqrt{2}$

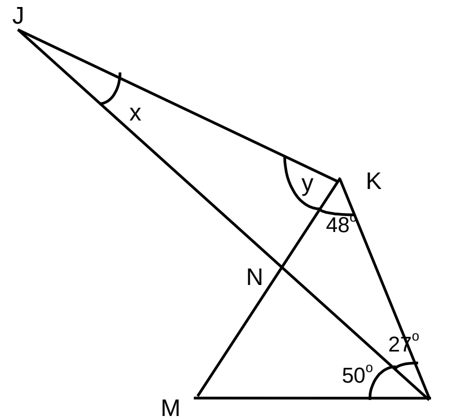
4. Lines AB,CD and EF in the figure above are parallel. Find the length of DF.



(a) $1\ ^3/_4$ (b) 3

(c) 2 (d) $2\ ^{1/2}$

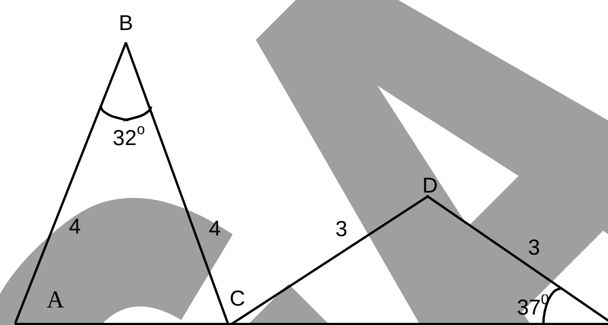
5. What is the sum of Angle x and Angle y?



(a) 55° (b) 75°

(c) 82° (d) 105°

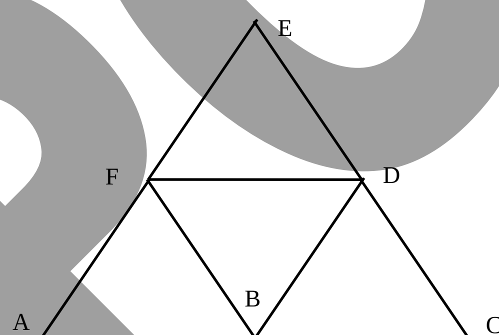
6. Using the information given in the diagram, what is the measure of Angle BCD?



(a) 37° (b) 69°

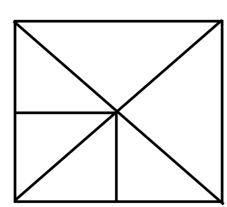
(c) 74° (d) 89°

7. In the figure, all triangles are equilateral. The area of AEC is 32. What is the area of trapezoid AFDC?



(a) 20 (b) 27 (c) 21 (d) 24

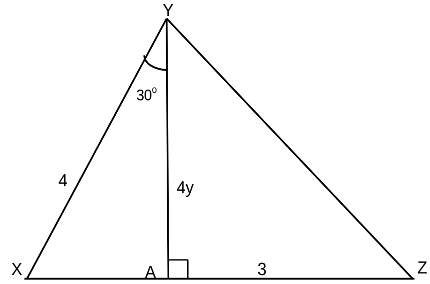
8. How many triangles are there in the figure?



(a) 10 (b) 11

(c) 12 (d) 15

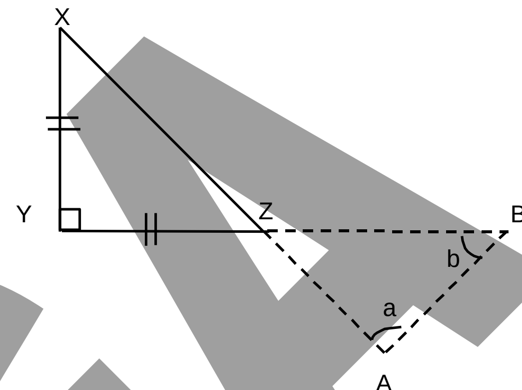
9. Find the area of the triangle XYZ in the given figure.



(a) 16 (b) 20y

(c) $5\sqrt{2}y^2$ (d) 10y

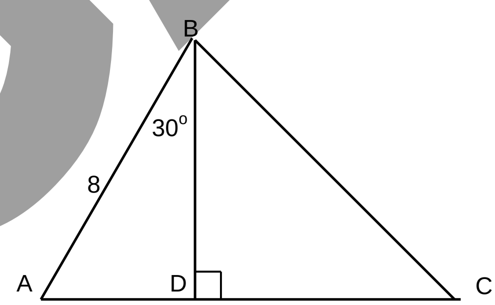
10. In the diagram, triangle XYZ is an isosceles right triangle. Sides XZ and YZ are extended through Z and another triangle constructed. What is a + b?



(a) 45° (b) 60°

(c) 85° (d) 135

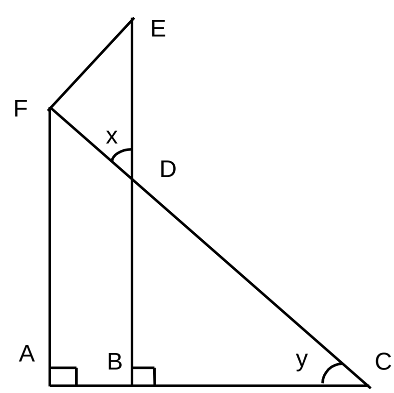
11. In the diagram below, what is the area of triangle ABC , given AC = 10?



(a) $20\sqrt{3}$ (b) $40\sqrt{3}$

(c) 60 (d) $9\sqrt{3}$

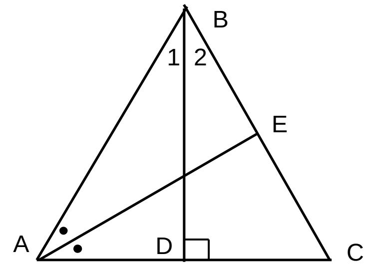
12. In the diagram, AF is parallel to BE and angle FAC is a right angle. What is the sum of x and y?



(a) 72° (b) 36°

(c) 90° (d) 120°

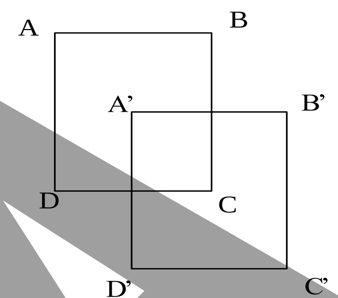
13. In the diagram, triangle ABC is equilateral. If BD is the altitude to side AC and AE is an angle bisector, which of the following statements is (are) true?



I. angle 1 = angle 2
II. AE = BD
III. AE is perpendicular to BC

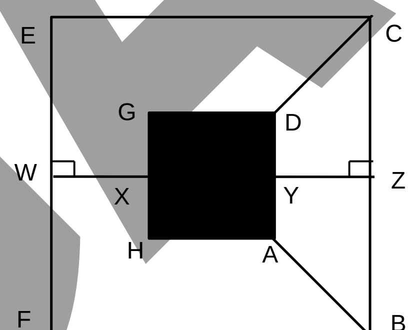
(a) I (b) II
(c) III (d) I, II and III

14. In the diagram, squares ABCD and A'B'C'D' are congruent. If AB = 8 and A' and C are the centers of the squares, find the area of the figure.



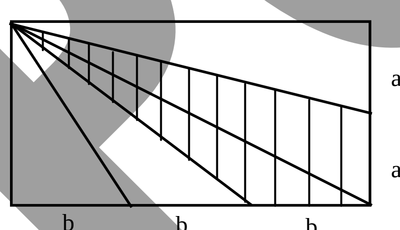
(a) 144 (b) 112
(c) 96 (d) 132

15. The area of shaded square ADGH is 4. The area of the unshaded portion of square BCEF is 21. AD is parallel to BC. WX equals YZ. What is the area of ABCD?



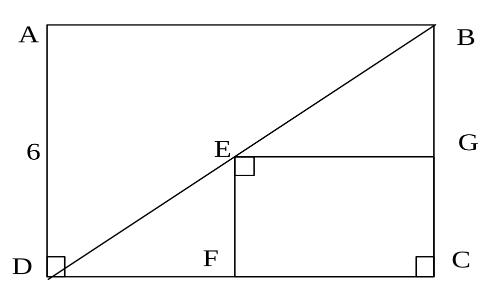
(a) 4 (b) $^{21}/_{3}$
(c) $^{21}/_{4}$ (d) $5\ ^{1}/_{2}$

16. In the rectangle in the diagram, what is the sum of the areas of the two shaded portions?



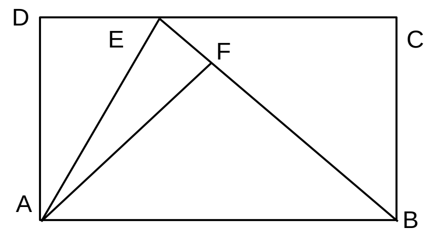
(a) 2ab (b) ab/2 (c) 3ab/2 (d) 5ab/2

17. In the figure, E is the midpoint of diagonal BD, AD is 6 and angle BDC is 30°. What is the area of a small rectangle EFCG?



(a) $6\sqrt{3}$ (b) $9\sqrt{3}$
(c) 27 (d) 18

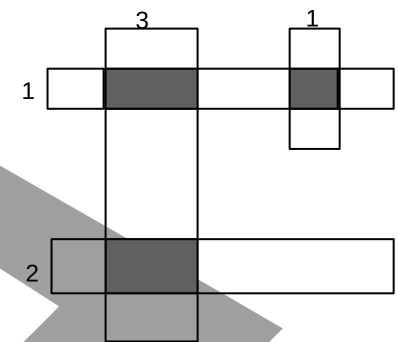
18. In the diagram, if AF = 15 and BE = 18, AF is perpendicular to EB. What is the area of Rectangle ABCD?



(a) 135 (b) 270

(c) 225 (d) 196

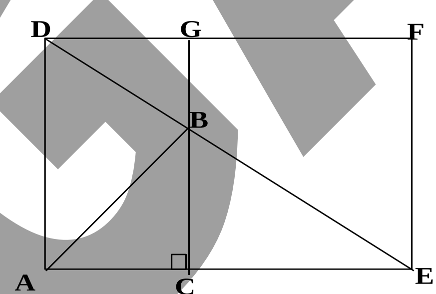
19. The following diagram shows a set of intersection rectangles. Find the area of the shaded region.



(a) 7 (b) 9

(c) 10 (d) 15

20. In the diagram, ADFE is a rectangle, DG = $^{1}/_{3}$ DF and BC = $^{2}/_{3}$ FE. What is the ratio of the area of triangle DEF to the area of triangle ABC?



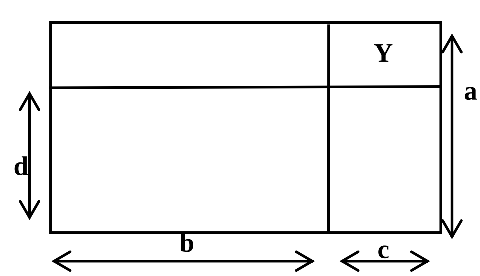
(a) 2 : 1 (b) 3 : 1

(c) 4 : 1 (d) 9 : 2

21. Perriton lies 6 miles south of Larburg. Larburg is 10 miles east of New Deburgh. Kauston is 3 miles north of New Deburgh. How far is Perriton from Kauston?

(a) √45 miles (b) √109 miles (c) √136 miles (d) √181 miles

22. Find an expression for the area of rectangle Y.



(a) bd - da (b) a² + cd

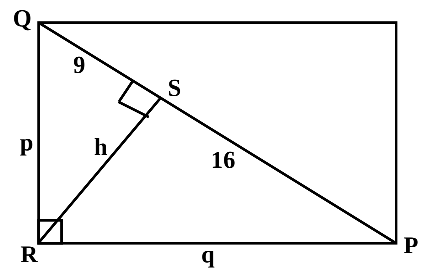
(c) ab + dc (d) ac - cd

23. A square of side s and an equilateral triangle of side s are both placed inside a rectangle of length 2s and width s. What fraction of the area of the rectangle remains uncovered?

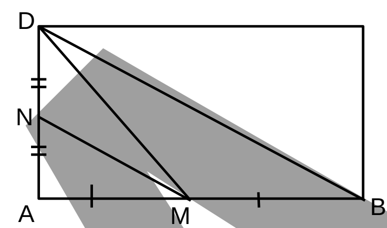
(a) (2+4√3)/8 (b) 1/2 +√3/4 (c) 1/2 - √3/4 (d) (4 - √3)/8

24. A sheet is in the form of a rhombus whose side is 8 metres and one of the diagonals is 10 metres. Find the cost of painting both the surfaces at the rate of Rs. 5 per sq. metre.

25. From the information given in the figure, find p, q and h

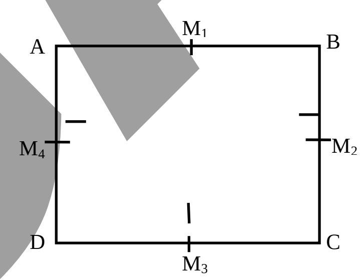


26. If area of the rectangle is 32 sq. cm . Find the area of the triangle AMN.



27. A circle of area 154 cm^2 is inscribed in an equilateral triangle. What is the ratio of the radius of the ex-circle to that of the incircle?

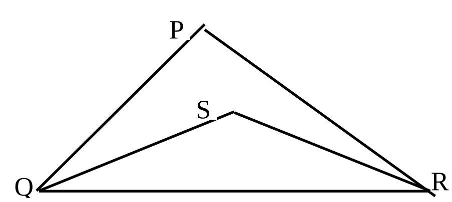
28. In the square ABCD, M_1, M_2, M_3 and M_4 are midpoints of the sides AB, BC, CD and AD respectively. If l(AB) = $(10/\sqrt{2})$ cm, find A(□$M_1M_2M_3M_4$).



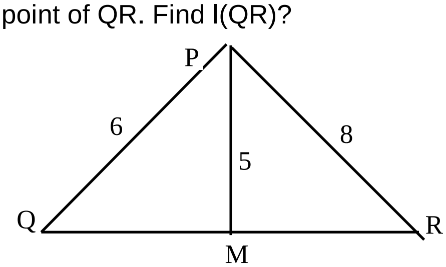
29. A ladder 50' long is leaning against a straight wall. Its lower end is 30' away from the base of the wall. If the lower end of the ladder is moved 10' further from the wall, at what height will the upper end of the ladder now rest against the wall?

30. In a circular pond, a fish starts from a point on the edge of the pond, swims 143 m due north, to reach another point on the edge, turns east and swims 24 m to reach yet another point on the edge. What is the diameter of the pond?

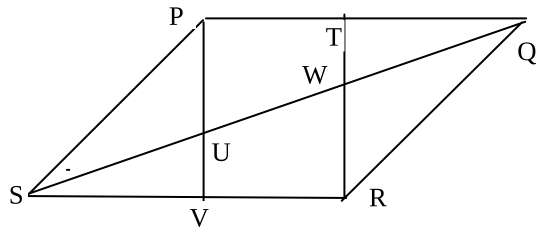
31. In the adjoining figure m∠QPR = 66°. QS and SR are the bisectors of ∠Q and ∠R respectively. Find m∠QSR.



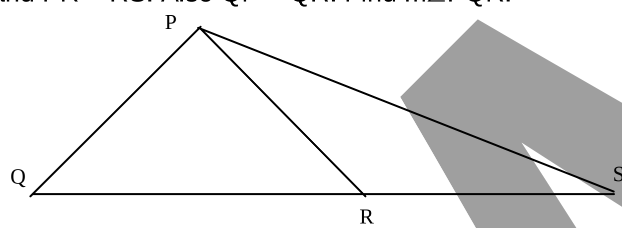
32. If M is the midpoint of QR. Find l(QR)?



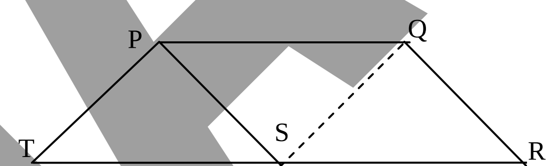
33. V and T are mid points of SR and PQ respectively. SQ = 15 cm is the diagonal of the parallelogram PQRS. Find l(UW)?



34. If m∠RSP 25° and PR = RS. Also QP = QR. Find m∠PQR.



35. In the adjoining figure there are two parallelograms named □ PQRS and□PTSQ. If the area of ΔPTS is 13 sq. units. Find the area of the trapezium TPQR?



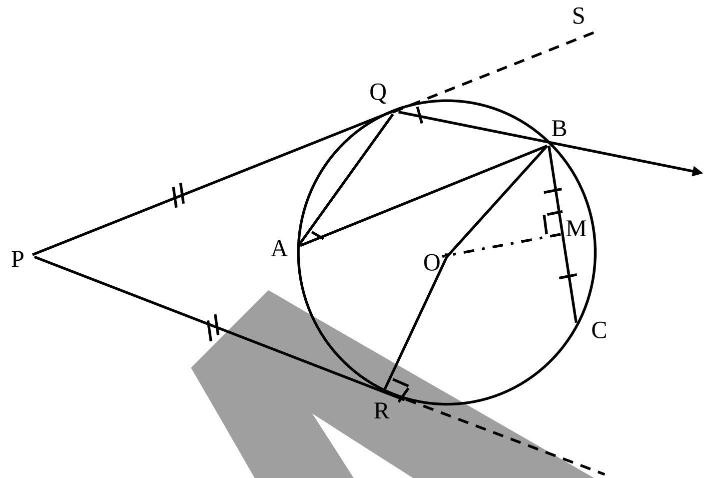
Answers

1. b	**2.** a	**3.** d	**4.** c
5. d	**6.** b	**7.** d	**8.** c
9. d	**10.** d	**11.** a	**12.** c
13. d	**14.** b	**15.** c	**16.** d
17. b	**18.** b	**19.**c	**20.** d
21. d	**22.** d	**23.** d.	**24.** Rs.100√(39)
25. q=20, p=15, h=12	**26.** 4	**27.** 2:1.	**28.** 25 cm^2.
29. 30’.	**30.** 145 m.	**31.** 123º.	**32.** 10 units.
33. 5 units.	**34.** 80º.	**35.** 39 sq. units.	

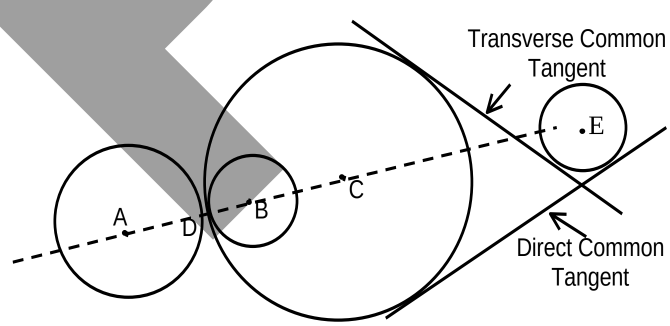
Concepts 3

Circle : Some Important properties :

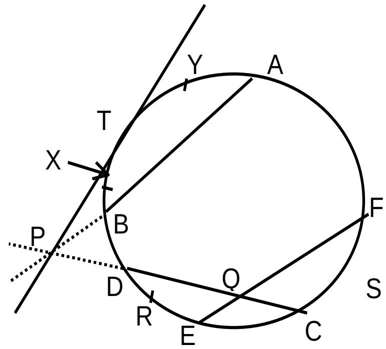
PQ = **PR** = Tangent.
O = Centre of the circle.
BC = Chord of the circle.
QB = Secant of the circle.
M = Mid point of chord BC.
OB = **OR** = Radius of the circle.
OM = Perpendicular to the chord BC.
AQB = Minor Arc.
ARB = Major Arc.



- Tangent is perpendicular to the radius i.e. PR is perpendicular to the radius OR.
- Perpendicular from the centre bisects the chord i.e. OM bisects BC.
- Tangent segments drawn from an external point are equal i.e. PR = PQ..
- Measure of an arc of a circle means the measure of the central angle i.e. m arc(AQB) = measure of the angle AOB.
- Angle made at the centre by an arc is equal to twice the angle made by the arc at any point on the remaining part of the circumference i.e. $\angle$ BOQ = 2 $\angle$ BAQ.
- Angles inscribed in the same arc are equal i.e. $\angle$ BAQ = $\angle$ BRQ = $\angle$ BCQ.
- The angle between a tangent and a secant at the point of contact is equal to the angle in the alternate segment i.e. $\angle$ BQS = $\angle$ BAQ



- When two circles touch, their centres and the point of contact are collinear i.e. A-D-B & D-B-C.
- If two circles touch externally, distance between their centres is equal to the sum of their radii i.e. AC = AD + DC
- If two circles touch internally, distance between their centres is equal to the difference of their radii i.e. BC = CD - BD.
- Length of Direct Common Tangent = $\sqrt{\{(d^2 - (R-r)^2\}}$
- Length of Transverse Common Tangent = $\sqrt{\{(d^2 - (R+r)^2\}}$

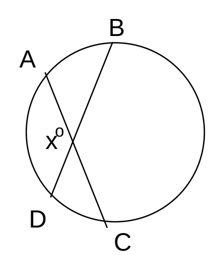


Where, R : Radius of circle with centre C.

And r : Radius of circle with centre E, and, d = Distance CE

If two chords intersect externally at P, (i) PA .PB = PC.PD (ii) $\angle P = {}^1/_2$ [m(arc AC) - m(arc BD)]

- If PBA is a secant and PT is a tangent, (i) PA .PB = PT^2 (ii) $\angle P = {}^1/_2$ [m(arc AYT) - m(arc BXT)]
- If chords EF & CD intersect internally at Q, then (i) QC.QD = QE.QF(ii) $\angle DQE = {}^1/_2$[m(arcFSC)+m(arc DRE)]
- m(minor arc) + m(major arc) = 360^o
- Measure of a semicircle = 180^o
- Equal chords of a circle are equidistant from the centre.
- Circles having the same centre but different radii are called concentric circles.
- The opposite angles of a cyclic quadrilateral are supplementary.
- Circumference of a circle = 2π r   Area = πr^2
- Area of a circular ring = $\pi (r_1 + r_2)(r_1 - r_2)$ r_1 = outer radius, r_2 = inner radius
- Distance covered by a wheel in n revolutions = n (circumference)
- length of an arc = $(\theta/360)$x 2π r where θ is the angle at the centre
- Area of sector = $(\theta/360)$x πr^2

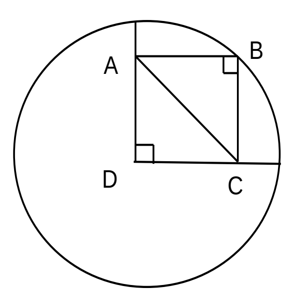


1. In the diagram, arc BC is 80° and arc AD is 40°. What is x?

(a) 80° (b) 70° (*c) 60° (d) 47°

Sol : The measure of an interior angle of a circle is found by adding the two intercepted arcs and dividing by 2. The angle x is therefore, $(80^o+40^o)/2 = 60^o$.

2. Given circle of radius r with a rectangle inscribed in one quadrant. What is the length of diagonal AC?

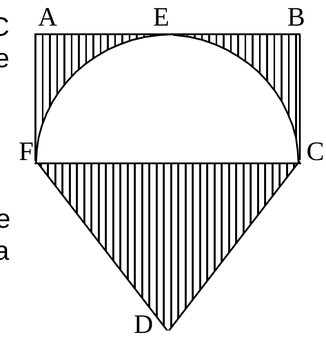


(a) $3\pi r^2/4$ (b) $2\pi r/3$ (c) $\pi r^2/4$ (*d) r

Sol : Simply draw the other diagonal (DB) of the rectangle. This is the radius of the circle. As diagonals of a rectangle are equal, diagonal AC = radius = r.

3. In the diagram, triangle EDC is equilateral and semicircle EFC is inscribed in rectangle ABCE. If side ED is 2, what is the area of the shaded portion of the figure?

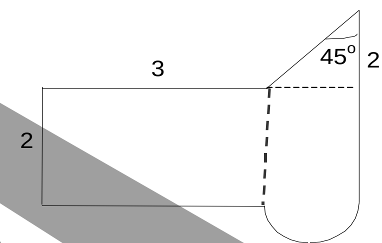
(a) $\sqrt{2} + 3 - 8\pi$ (b) $\sqrt{3} + 4 - 3\pi$
(c) $\sqrt{5} + 3 - 16\pi$ (*d) $\sqrt{3} + 2 - {}^{1}/_{2}\,\pi$



Sol : ED = 2, the area of the equilateral triangle is $4\sqrt{3}/4 = \sqrt{3}$. The rest of the shaded portion can be found out by taking the area of the rectangle and subtracting from it the area of the circle.
$\therefore$The total area then becomes $\sqrt{3} + 2 - 0.5\pi$

4. What is the total perimeter of the figure given below?

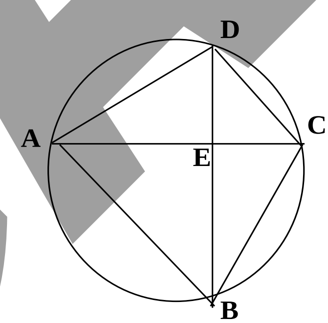
a) $14 + 3\sqrt{2} + 2\pi$ (*b) $12 + 2\sqrt{2} + \pi$
(c) $3 + 12s\sqrt{3} + 4\pi$ (d) $7 + 1\sqrt{2} + \pi$



Sol : Add up the component parts. Semicircular perimeter is π. Hypotenuse of the isosceles right triangle is $2\sqrt{2}$. Total of horizontal and vertical lines is 12.
$\therefore$ Total perimeter $= 12 + 2\sqrt{2} + \pi$

5. In the diagram, if quadrilateral ABCD is inscribed in the circle, which triangle is similar to triangle BEC?

(a) triangle ABC (*b) triangle ADE
(c) triangle DBC (d) triangle ADB

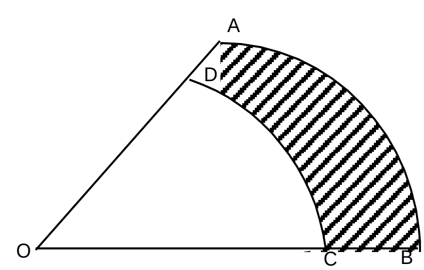


Sol : Two triangles are similar if 2 of their corresponding sides are equal. Angle AED = angle BEC.
Angle ADB = Angle ACB, inscribed angles of the same arc. triangle ADE similar to triangle BEC.

Exercises

1. In the diagram, AOB and COD are sectors of circles with center O. If DO = 4, AO = 5 and DOC = 45°, what is the area of the shaded region?

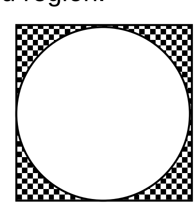
(a) ${}^{9}/_{8}\,\pi$ (b) π
(c) 9π (d) ${}^{\pi}/_{3}$



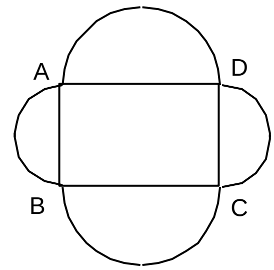
2. If the area of the square is 1, find the area of the shaded region.

(a) $1 + {}^{\pi}/_{2}$ (b) $1 - {}^{\pi}/_{4}$

(c) $2 - {}^{\pi}/_{2}$ (d) $1 - {}^{\pi}/_{6}$



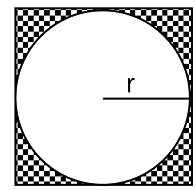
3. In the diagram, find the area of the figure if AB = 6, AD = 8 and the sides of the rectangle are diameters of the semi-circles.



(a) $48 + 25\pi$ (b) $36 + 10\pi$

(c) $48 + 14\pi$ (d) 36

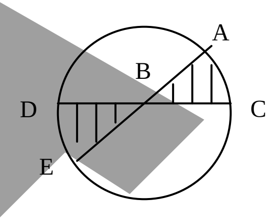
4. Find an expression for the shaded area shown in fig.



(a) $(r + \pi)^2$ (b) $r^2 - 2\pi$

(c) $2r^2 - \pi r^2$ (d) $r^2(4-\pi)$

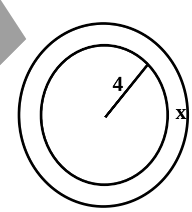
5. The area of the entire circle below is 63. The area of the shaded portion is 14. What is the number of degrees in angle ABC if B is the centre of the circle?



(a) 40 (b) 45

(c) 80 (d) 30

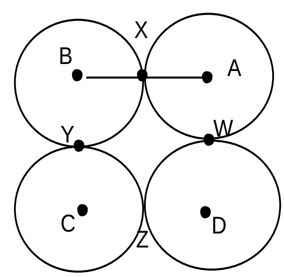
6. The diagram below depicts two concentric circles forming a ring. The inner radius is 4 and the area of ring is 176. Find the thickness, x. (Use $^{22}/_7$ for π)



(a) $6\sqrt{2} - 4$ (b) 52

(c) $\sqrt{56}$ (d) 3

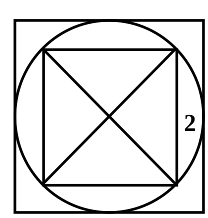
7. In the diagram, all the circles are of equal size. A, B, C, and D are the centers of the circles. If AB is one side of a square then which of the following is another side of the square?



(a) XY (b) BC

(c) BD (d) YW

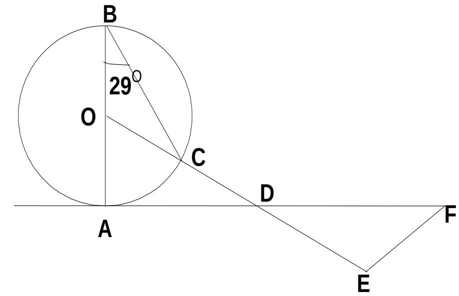
8. The diagram, shows a circle with an inscribed and a circumscribed square. The side of the smaller square is 2. What is the side of the larger square?



(a) 3 (b) $2\sqrt{2}$

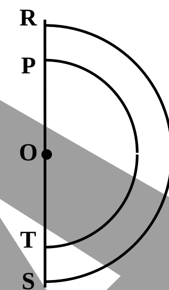
(c) $3\sqrt{6}$ (d) $\sqrt{8} + 1$

9. In the diagram; O is the center of the circle. Angle ABC is an inscribed angle measuring 29°. Angle AOC is a central angle. Line AF is tangent to the circle at point A. Angle EDF measures



(a) 15° (b) 29°
(c) 32° (d) 58°

10. Point O is the common center for the half-circle with a radius of OP and for the half circle with a radius of OR. OR = 3/2 OP. S,T,O,P, and R all lie on the same line. What is the difference between the measure of arc RS and arc PT?

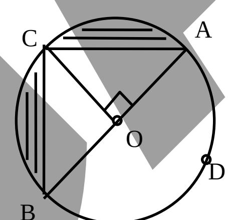


(a) 0° (b) 30°
(c) 60° (d) 90°

11. What is the area of the shaded area if Arc ADB = 180°, Arc AC = 90°, and CB = 4?

(a) 4π - 8 (b) 8π - 8

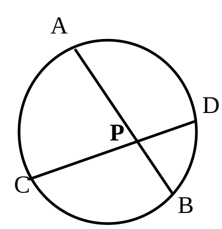
(c) 8π - 16 (d) 4π



12. What is the area of the circle inscribed in a square with diagonal $14\sqrt{2}$ cms. ?

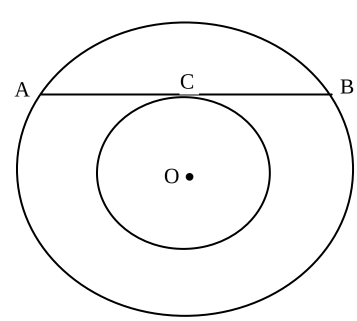
13. The distance between the centres of two circles is 17 cm. and their radii are 12 and 4 cm respectively. Find the length of the direct common tangent to the circles. Find the length of the transverse common tangents.

14. If CD = 9 cm, PD = 3 cm, PA = 2 cm find PB.

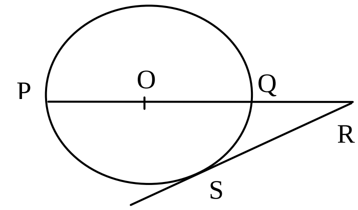


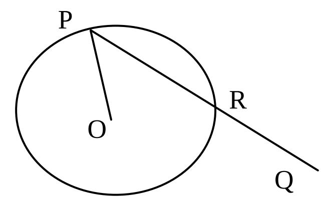
15. A horse is tethered at one corner of a square plot of side 42 m by a rope 35 m long. Find the ungrazed area.

16. The line AB, 8 cm in length, is tangent to the inner circle at the point C. If the area of Δ OAB is 12 cm^2, what is the radius of the outer circle?



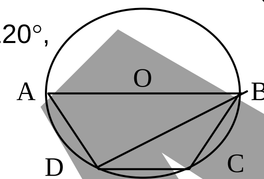
17. RS is tangent to the circle at point 'S'.If 'O' is the centre of the circle and l(SR) = 12 units and l(QR) = 9 units. Find the radius of the circle?





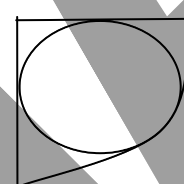
18. OP = 9 units is the radius of the circle. If PR = 6 units and RQ = 4 units, find OQ?

19. 'O' is the centre of the circle. If m∠BCD = 120°, Find m∠ABD?

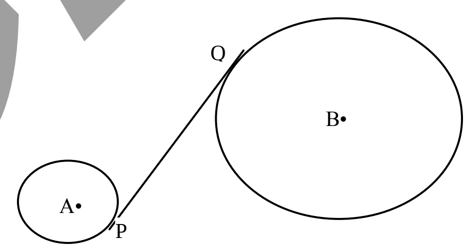


20. A rectangle is inscribed in a circle of radius 6.5 cm. If one side of the rectangle is 12 cm, find the area of the rectangle.

21. A circle is inscribed in a quadrant circle as shown in the figure. If the radius of the quadrant circle is R, what is the radius of the inscribed circle?



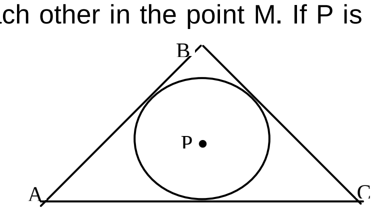
22. Line PQ is tangent to the two circles at P and Q. A and B are the centres of the two circles with radii 3 cm and 5 cm respectively. Find the ratio in which PQ divides the segment AB.



23. A square has two of its vertices on a circle and the other two on a tangent to the circle. If the diameter of the circle is 10 cm, what is the area of the square?

24. Chords AB and CD of a circle, with radius 13 cm, intersect each other in the point M. If P is the centre of the circle and l(PM) = 5 cm, find (CM) x (DM).

25. In the adjoining figure, m∠ABC = 90°, l(AB) = 6 inches and l(BC) = 8 inches. What is the area of the incircle of ΔABC?



Answers

1. a.	2. b.	3. a.	4. d.	5. a.
6. a.	7. b.	8. b.	9. c.	10. a.
11. a.	12. 49π.	13. 15 cm^2, $\sqrt{33}$ cm.	14. 9 cm.	15. 801.5 m^2.
16. 5 cm.	17. 3.5 units.	18. 11.	19. 30º.	20. 60 cm^2.
21. $R(\sqrt{2} - 1)$.	22. 3:5.	23. 64 cm^2.	24. 144 cm^2.	25. 4π.

Concepts 4

Co-ordinate Geometry

INTRODUCTION :

In figure 1, X'OX and YOY' intersect each other at right angle in point O. The plane is divided in four quadrants. Positive real numbers are represented on rays OX and OY with number 0 at O. Point O is called the origin, X'OX the x-axis and Y'OY the y-axis.

Consider Point P in the first quadrant. Draw segment PN perpendicular to OX and PM perpendicular to OY. Point N represents a positive number (x_1) on x-axis.

Similarly, point M represents a number (y_1) on positive y-axis. The number x_1 is called the x co-ordinate or **abscissa** of the point P and y_1 is called the y co-ordinate or ordinate of the point P. (x_1, y_1) together are called the co-ordinates of the point P and the point is denoted by P ≡ (x_1, y_1). The following table shows the nature of values of x and y coordinates in the four quadrants formed by the two co-ordinate axes.
X axis is known as the **real** axis and Y axis is known as the **imaginary** axis.

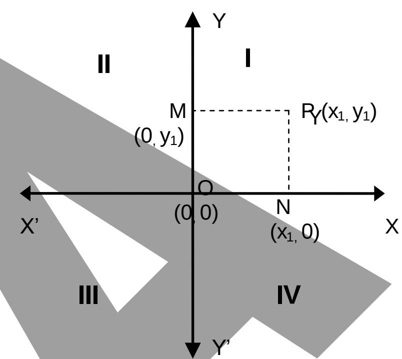


Figure 1. Coordinate Axes

Co-ordinates	Quadrant I.	Quadrant II.	Quadrant III.	Quadrant IV.
X	+ ve	- ve	- ve	+ ve
Y	+ ve	+ ve	- ve	- ve

DISTANCE FORMULA: Distance between two points P(x_1, y_1) and Q (x_2, y_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

From the distance formula, it can be shown that the distance of a point P(x_1, y_1) from origin (0, 0) is given by:

$$OP = \sqrt{x_1^{\,2} + y_1^{\,2}}$$

Ex.. Find the distance between the two points A(3, 2) and B(7, 5)

Sol.

$$AB = \sqrt{(7-3)^2 + (5-2)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25} = 5$$

Ex. Find y_2 if the distance between P(4, 8) and Q(6, y_2) is √68

Sol. PQ = √(68) = √{(6 - 4)^2 + (y_2 - 8)^2}

= √{4 + (y_2 - 8)^2}

Squaring both the sides,
68 = 4 + (y_2 - 8)^2
(y_2 - 8)^2 = 64 ∴y_2 - 8 = ± 8, ∴y_2 = 16 or 0.

SECTION FORMULA: Co-ordinates (x, y) of the point P which divides the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m :n are given by the following section formula:

AP = m

BP = n

$x = (m x_2 + n x_1) / (m + n)$

$y = (m y_2 + n y_1) / (m + n)$

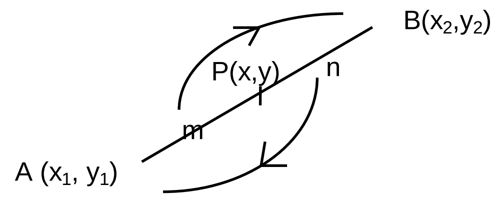


Figure 2.

In case of external division, change n to -n. If the ratio m : n = m/n is written as k, (i.e., m/n = k) then the section formula takes the form:

$x = (k x_2 + x_1) / (k + 1)$ and $y = (ky_2 + y_1) / (k + 1)$

Ex. Find the co-ordinates of point P which divides the line AB joining A(3, 1) and B (7, -5) externally in the ratio 3 : 2.

Sol. Since the division is external, m = 3 and n = -2 (Refer figure 3)

$x =[3 \times 7 + (-2 \times 3)] / [3 + (-2)]$

$y =[3 \times (-5) + (-2) \times 1] / [3 - 2]$

x = 15 y = -17

P = (15, - 17)

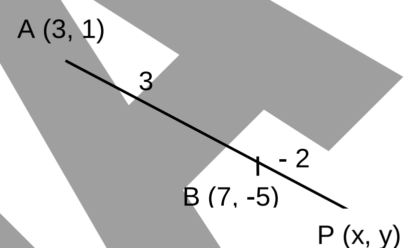


Figure 3.

Ex. Find the ratio in which point A (12, 13) divides the join of P (- 2, 1) and Q (5, 7).

Sol. Let A divide segment PQ in the ratio k : 1 then by section formula, we obtain the following equation

$(12, 13) \equiv ([5k -2]/[k + 1], [7k + 1]/ k + 1)$; $12 = (5k - 2) /(k + 1)$; $12k + 12 = 5k – 2$; so $k = -2$.

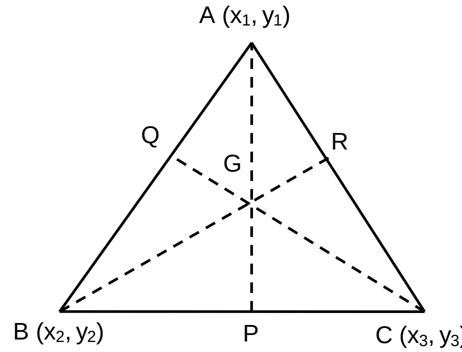
Therefore, A divides the join of P and Q externally in the ratio 2 : 1

Note: From section formula, it can be noted that the mid-point of the segment joining $A (x_1, y_1)$ and B (x_2, y_2) is given by:

$x = (x_1 + x_2) / 2$ and $y = (y_1 + y_2) / 2$

TRIANGLE :

Median is a segment joining mid-point of a side of a triangle and the opposite vertex. In the adjoining figure (Fig. 4) the three medians of triangle ABC are AP, BR and CQ. The point where the three medians intersect each other (G) is called the centroid of the triangle. Centroid divides the medians in the ratio 2 :1.



Medians AP, BR, CQ

Centroid G, AG: GP :: 2 :1

Figure 4

Centroid G =[(x_1 + x_2 + x_3) / 3, (y_1 + y_2 + y_3) / 3)]

Area of the triangle = 1/2 [$x_1 \times (y_2 - y_3) + x_2 \times (y_3 - y_1) + x_3 \times (y_1 - y_2)$]

If vertices are taken in clockwise direction area of the triangle comes to be negative , otherwise it is positive. This can be written in the determinant form as follows

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Examples

5. P (-3, 4) is the centroid of a triangle whose vertices are (0, y), (6, 2) and (x, 3). Find x and y.

From the formula for centroid of the triangle,

- 3 = (0 + 6 + x) / 3; - 9 = x + 6; so x = - 15
Similarly,
4 =(y + 2 + 3) / 3; 12 = y + 5; y = 7

6. Find area of a triangle whose co-ordinates are (5, 2) , (-9, -3), and (-3, -5).

Area = [1/2] × [5 × (-3 + 5) + (-9) × (-5 - 2) + (-3) × (2 + 3)]
= [1/2] × [10 + 63 - 15]
= 58/2 = 29.

Note: 1. Area of triangle is zero when the given three points are collinear.
2. Area of a quadrilateral ABCD can be considered as the sum of the areas of two triangles, ABC and ACD

7. Show that the three points (2, -4), (10, 0) and (12, 1) are collinear.
If the area of the triangle formed by the given three points is zero then the points are collinear.
Area = 1/2 [2 × (0 - 1) + 10 × (1 + 4) + 12 × (-4 - 0)]
= 1/2 [-2 + 50 - 48]
= 0, thus the points are collinear.

8. Co-ordinates of the two vertices of the triangle are (2, 4) and (4, -2). The coordinates of the centroid are (5/3, 5/3). Find the coordinates of the third vertex.

Let $(x_1, y_1) = (2, 4)$; $(x_2, y_2) = (4, -2)$ and G (x, y) = (5/3, 5/3). We have to find (x_3, y_3).

Centroid G = $[(x_1 + x_2 + x_3) / 3 ; (y_1 + y_2 + y_3) / 3]$. $\therefore (x_1 + x_2 + x_3) / 3 = 5/3$ and $(y_1 + y_2 + y_3) / 3 = 5/3$, $\Rightarrow 6 + x_3 = 5$ and $2 + y_3 = 5$, $\therefore x_3 = -1$, $y_3 = 3$. So the co-ordinates of third vertex are (-1, 3).

POINTS OF THE LOCUS:

If a point satisfies the given condition or given equation then it is said to be on the locus.

Examples

9. P (k, 3) is a point on the locus whose equation is $5x + 4y = 14$. Find k.

$5 \times k + 4 \times 3 = 14$; $5k = 2$; $k = 2/5$

10. Find the co-ordinates of point of intersection of two lines given by the equations, $3x + 4y = 7$ and $5x - 4y = 9$.

A point of intersection of two loci can be obtained by solving the given equations simultaneously.

$$\begin{array}{rl} & 3x + 4y = 7 \\ + & \underline{5x - 4y = 9} \\ & 8x \quad = 16 \end{array}$$

$x = 2 \qquad y = {}^1/_4$

The point of intersection of the above two lines is therefore (2, 1/4)

11. Show that the point (2, -2) lies on the locus whose equation is $x^2 + 4y^2 = 10x$ and that the point (-1, 2) does not lie on the locus.

The given equation is: $x^2 + 4y^2 = 10x$ . . . . . (1)
Consider (2, -2), substitute x = 2 in equation (1)

$4 + 4y^2 = 10 \times 2 = 20$

$y = \pm 2$

Since (2, -2) satisfies the given equation of the locus, it can be said that (2, -2) lies on the locus.
Consider (-1, 2), substitute x = -1 in equation (1)

$1 + 4y^2 = 10 \times (-1)$

$\Rightarrow 4y^2 = -11$, $\Rightarrow y^2 = -11/4$, $\therefore$ y is imaginary.

The point (-1, 2) does not satisfy the equation of the locus, it does not lie on the locus.

LINE:
Angle (θ) made by a line or a ray with positive side of the x-axis is called the inclination of the line.

Inclination of x-axis and any line parallel to x-axis is zero.
Inclination of y-axis and any line parallel to y-axis is 90^0

$\tan\theta$ is the slope of the line. The slope is generally denoted by m $\therefore$ Slope of the line = m = $\tan\theta$
If θ is acute the slope is positive, If θ is obtuse, the slope is negative.
Slope of x-axis or any line parallel to x-axis is zero.

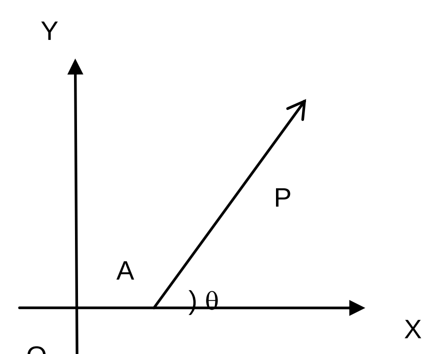


Figure 5.

Slope of y-axis or any line parallel to y-axis is infinite.
Slope of a line passing through (x_1, y_1) and (x_2, y_2) is given by:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

When two lines whose slopes are m_1 and m_2 are **parallel**, then $\mathbf{m_1 = m_2}$ (or, their slopes are equal).
When the two lines are **perpendicular** to each other, the product of their slopes is -1
i.e. $\mathbf{m_1 \times m_2 = -1}$

Examples

12. Find the slope and inclination of the lines. Given that the points (-2, 2) and (-8, -4) both lie on the same line

    Slope = $m = (y_2 - y_1) / (x_2 - x_1) = (-4 - 2) / (-8 + 2) = 1$
    Slope = $\tan \theta = 1$; $\theta = 45^0$. Thus the inclination is 45 degrees..

13. A line containing $(x_1, 2)$ and (5, 4) has a slope 1/4. Find x1. Is this line perpendicular to the line segment whose end points are A (0, 2) and B (8, 4).
    $m_1 = 1/4 = (4 - 2) / (5 - x_1)$ i.e., $5 - x_1 = 8$ or, $x_1 = -3$
    Slope of the line segment AB = $m_2 = (4 - 2) / 8 = 1/4$
    Since the slopes are equal the lines are parallel.

Equation of a Line:

1. Equation of x-axis $y = 0$
2. Equation of any line parallel to x-axis $y = k$ (k is a constant)
3. Equation of y-axis $x = 0$
4. Equation of any line parallel to y-axis x = constant

Equation of a line can be obtained in several different forms as indicated in the following table.

Type	Information Needed	Standard Form
1.Slope - Point Form	Co-ordinates of point A (x_1, y_1) Slope (m) of the line.	$(y - y_1) = m (x - x_1)$
2.Slope Intercept Form	Slope of the line = m and Y intercept = c	$y = m x + c$
3.Two Points form	Co-ordinates of points A (x_1, y_1) and B (x_2, y_2).	$(x - x_1) / (x_2 - x_1) = (y - y_1) / (y_2 - y_1)$
4.Two Intercept form	X intercept a, Y intercept b)	$(x/a) + (y/b) = 1$
5.Point inclination form	Inclination = θ, Point =(x_1, y_1)	$(x - x_1) / \cos \theta = (y - y_1) / \sin \theta$
6.Normal form	p = perpendicular distance from origin, α it's inclination	$x \cos \alpha + y \sin \alpha = p$
7.General equation	Slope = **- a/b**, X-intercept **= - c /a,** Y- intercept = **- c/b.**	$ax + by + c = 0$

Examples

14. Find the equation of a line whose slope is 0.5 and passes through a point (3, 2).

    The equation of a line in slope - point form is: $y - y_1 = m (x - x_1)$.
    Substituting, $m = {}^1/_2$, $x_1 = 3$ and $y_1 = 2$
    $y - 2 = {}^1/_2 (x - 3)$
    $\therefore$ $2y - 4 = x - 3$, $\therefore$ Required equation is $2y = x + 1$.

15. A line inclined at 45 degrees to positive x-axis intersects y-axis at (0, 3). Find the equation of the line.

Slope of the line = tan θ (θ is the inclination) = tan 45^0 = 1. Equation of a line in slope - intercept form is y = m x + c, where, c is the y - intercept and m is the slope. y = 1 $\times$ x + 3, x -y + 3 = 0.

16. Points P (2, 5) and Q (-3, 2) both lie on the same line. Find the equation of the line.

Equation of a line passing through A (x_1, y_1) and B (x_2, y_2) is: (x - x_1)/(x_2 - x_1) = (y - y_1) / (y_2 - y_1)
For the given line, equation is: (x - 2) / (-3 -2) = (y - 5) / (2 - 5). (x - 2) / -5 = (y - 5) / -3
- 3x + 6 = 25 - 5y, 3x - 5y + 19 = 0

Angle Between Two Intersecting Lines:

If L_1 is a line given by: y = m_1 x + c_1 and L_2 is given by: y = m_2x + c_2. Then the acute angle (θ) between these two lines can be found by using the following equation:

$$\tan\theta = \left| [(m_1 - m_2) / [1 + (m_1 \times m_2)] \right|$$

Example

17. Find the acute angle between 3x - 4y + 7 = 0 and 2x + 14 y = 7

Consider 3x - 4y + 7 = 0
y = (3/4) x + (7/4) m_1 = 3/4
Consider 2x + 14y = 7
y = (-2/14)x + (7/14)
y = (-1/7)x + (1/2) m_2 = -1/7
Let θ be the acute angle between the two lines L_1 and L_2.

tan θ = [(m_1 - m_2) / [1 + ($m_1 \times m_2$)] = [(3/4) - (-1/7)] / [1 + (3/4) $\times$ (-1/7)]
tan θ = [(21 + 4) / 28] $\div$ [(28 - 3) / 28] = (25 / 28) $\div$ (25 / 28) = 25 / 25 = 1
$\theta = 45^0$. The angle between the given lines is 45 degrees.

Distance Between Parallel Lines :

The distance between two parallel lines L_1 and L_2 whose equations are in the form:

ax + by + c_1 = 0 and ax +by + c_2 = 0 is given by $\left| (c_1 - c_2) / \sqrt{(a^2 + b^2)} \right|$

The length of perpendicular from point P(x_1 , y_1) to the line ax + by + c = 0 is given by

$$\left| (ax_1 + by_1 + c) / \sqrt{(a^2 + b^2)} \right|$$

Examples

18. Find the distance between the parallel lines 3x - 8y + 4 = 0 and 4y = 4 + (3/2)x

Given lines are 3x - 8y + 4 = 0 and 3x - 8y + 8 = 0
The distance between these two lines is...
$\left|(8 - 4) / \sqrt{(9+64)}\right| = 4 / \sqrt{73}$.

Some Important Results

1. A point (x_1, y_1) is $(x_1{}^2 + y_1{}^2)^{1/2}$ away from the origin.

2. The coordinates of any point on the join of (x_1, y_1) and (x_2, y_2) can be taken as [($\lambda x_2 + x_1$) / (λ + 1) **,** ($\lambda y_2 + y_1$) / (λ + 1)]. This point divides the given line in the ratio λ : 1.

3. Three points are collinear if the area of the triangle formed by them is zero.

4. **Coordinates of some standard points:**

(i) **Centroid of a triangle:** This is the point of intersection of the medians (i.e. the line joining a vertex to the mid point of the opposite side). This point divides each median in the ratio 2:1. Its co-ordinates are [($x_1 + x_2 + x_3$) / 3 , ($y_1 + y_2 + y_3$) / 3].

(ii) **Circumcentre of triangle:** This is a point which is equidistant from the three vertices of the triangle. It is also the point of intersection of right bisectors of the sides of the triangle (i.e. the lines through the mid point of a side and perpendicular to it). It is the centre of the circle that passes through the vertices of the triangle.

(iii) **Incentre of a triangle:** This is the centre of the circle which touches the sides of a given triangle. It is the point of intersection of the internal bisectors of the angles of the triangle. Its coordinates are given by the formula
x = ($ax_1 + bx_2 + cx_3$) / (a + b + c), y = ($ay_1 + by_2 + cy_3$) / (a + b + c), where a, b and c are the sides of the triangle.

(iv) **Orthocentre of a triangle:** This point is the intersection of the altitudes, (i.e. the lines through the vertices and perpendicular to opposite sides).

SOLVED EXAMPLES

1. A line with slope -1/2 intersects x-axis. A point Q(3, -5) lies on the same line. Find the point of intersection between the given line and the x-axis.
Let the line intersect x-axis at point P(x, 0). Q(3, -5) lies on the given line and the slope of the line is given to be -1/2.
slope = (-1/2) = [0 - (-5)] / [x - 3]; 3 - x = 10; x = - 7
Thus the point of intersection is P(-7, 0).

2. Write the equation of a line having inclination 60^0 and passing through the point (1, 2).
Inclination = 60^0 thus, slope = tan 60 = $\sqrt{3}$. Since this line passes through (1, 2) its equation can be written in the point slope form as (y - 2) = $\sqrt{3} \times$ (x - 1).

3. Find the slope and intercepts of the line 3x - 2y = 12, Also find the co-ordinates of the points where the line intersects the two coordinate axes.
Equation of the line is 3x -2y = 12 i.e. 2y = 3x -12 or, y = (3/2)x - 6
Comparing this with y = m x + c,
slope of the line = 3/2 and Y - intercept = - 6
On X - axis since y = 0, Putting y - 0 in the given equation we get X - intercept = 4
X- intercept = 4 P = (4, 0)
Y-intercept = -6 Q = (0, -6)

4. Show that the lines x - 4y + 8 = 0 and 8x + 2y = 7 are perpendicular to each other.
Comparing with ax + by + c = 0, the slopes of the given lines are - a/b = 1/4 = m_1 and m_2 = - 8/2 = - 4. $\therefore$ $m_1 \times m_2$ = -1; Since $m_1 \times m_2$ = -1, the given lines are perpendicular to each other.

5. Find equation of a line whose intercepts are opposite in sign compared to those of the line represented by following equation. $3x - 2y + 18 = 0$.
To find intercepts of $3x - 2y + 18 = 0$.
The X intercept is obtained by substituting $y = 0$ in the above equation. $3x + 18 = 0$; $x = -6$
The Y intercept is obtained by substituting $x = 0$ in the given equation. $-2y + 18 = 0$; $y = 9$
Intercepts of the required line are: 6, -9
The equation of line in two intercept form can be written as: $(x/6) + (y/-9) = 1$

6. Find the distance of line $3x - 4y = 15$ from the origin.

Note: Distance of a line $ax + by + c = 0$ from a point $P(x_1, y_1)$ is $\left| ax_1 + by_1 + c \,/\, \sqrt{(a^2 + b^2)} \right|$

Distance from origin = $\left| [\,3(0) - 4(0) - 15\,] \,/\, \sqrt{(9 + 16)} \right| = 15/5 = 3$

EXERCISES

1. Line intersects x-axis in A and y-axis in B. If A (10,0) and B (0, 10) find the equation of the line.

2. Find the slope of the line passing through (- 3, 7) having Y intercept -2. Also find its equation.

3. The coordinates of four points PQRS are P (0, -3) Q (6, 1) R (-4, -4) and S (5, 2). Show that the line segments PQ and RS are parallel to each other.

4. Find the acute angle between the two lines given by $3x - y + 4 = 0$ and $2x + y - 3 = 0$.

5. Find the equation of a line containing point (4, 5) and parallel to y axis. Also write down the equation of a line perpendicular to y-axis and containing the point (5, -6).

6. Prove that the triangle PQR having the three coordinates P (-2, 2) Q (4, 5) and R $(3, 2+2\sqrt{5})$ is an isosceles triangle.

7. What kind of a quadrilateral is formed by the vertices (0,0), (4,3), (3,5), (-1,2).

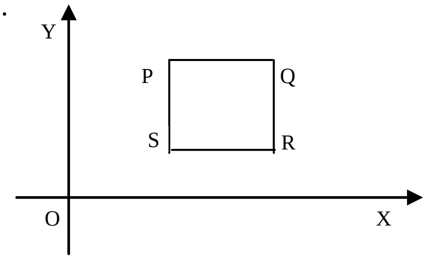
8. A(a, 0) and B(3a, 0) are the vertices of an equilateral triangle ABC. What are the coordinates of C ?

9. A triangle has its 12 units base on the line $3x + 7y = 12$. If the third vertex is at (3, -5), find the area of the triangle.

10. Find the area enclosed by the figure $|x| + |y| = 4$.

11. The co-ordinates of the vertices A and B are (6, 0) and (0, - 8) respectively. What is the area of the square ABCD?

12. The side PS of a square PQRS is parallel to the Y axis as shown in the adjoining figure. Calculate the slope of the diagonal SQ.



13. If points A(2,5), B(-7,2) and C(a,3) are collinear, find the co-ordinates of C.

14. Find the equations of the line passing through (-2,4) and having equal intercepts on the X-axis and the Y-axis.

15. Find the area of the triangle whose vertices are midpoints of AB, BC and CA. It is given that the coordinates of points A,B and C are (4,8),(20,6)and (-2,10).

16. Two parallel sides of a trapezium are of lengths 6 cm and 2 cm and lie on the lines represented by 3x + 4y = 15 and 6x + 8y = 20. Find the area of the trapezium?

17. If C is the centroid of the triangle PQR. If the coordinates of points P,Q, R and C are (x,1), (0,y), (1/2,0) and (1/2,1/3) respectively. Find m<PQR?

18. Find the inclination and perpendicular distance of a line represented by x/2 + √3 y/2 = 12√5, from the origin.

19. Segment AB is divided into five equal parts at P, Q, R and S. If the coordinates of P and R are (8,12) and (4,16). Find the equation of the line passing through 'S' and having an inclination of 135°.

20. What is the equation of the line parallel to the line x + 3y = -7 and passing through the centroid of the triangle formed by the intersection of the lines 3x - 4y = -11, 3x - y = - 5 and 3x + 2y = 19?

ANSWERS

1. x + y = 10.	**2.** -3, 3x + y + 2 = 0	**3.** Hint: Find the slopes of two lines.
4. 45^0	**5.** x = 4, y = - 6	**6.** Hint : Find $(PQ)^2$ $(QR)^2$ and $(PR)^2$
7. Parallelogram.	**8.** (2a, ± a√3)	**9.** 228 / (√58) sq. units
10. 32 sq. units.	**11.** 100.	**12.** 1.
13. – 4.	**14.** X+Y=2.	**15.** 10.
16. 4 cm^2.	**17.** 45º.	**18.** 60^o ,12√5 units.
19. x+y=20.	**20.** x + 3y = 16.	

Concepts 5

Trigonometry

Measuring Angles : In the hexadecimal system, the angles are measured in degrees, minutes and seconds. One complete rotation = 360 degrees (360^0**)**
1^0 = 60 minutes (60'); 1' = 60 seconds (60'')

In the circular system, the angles are measured in radians. π radians (π^c) = 180^0

Change From	T0	Multiply By
Radians	Degrees	180/ π
Degrees	Radians	π/180

Let S = length of arc A x B
θ = angle AOB expressed in radians
r = radius of the circle
Then, S = $r \times \theta$
And the area of sector 0 - A x B is A = $(1/2) \times r^2 \times \theta$

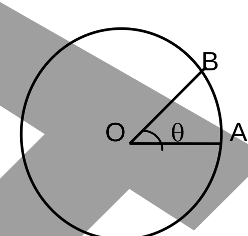
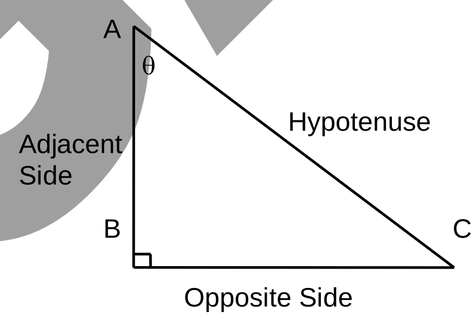


Figure x

Trigonometric Ratios: Consider the right angle triangle ABC as shown.



Sine θ = Opposite Side / Hypotenuse
= BC / AC

Cosine θ = Adjacent Side / Hypotenuse
= AB / AC

Tangent θ = Opposite Side / Adjacent Side
= BC / AB

cosec θ = 1 / sin θ
sec θ = 1 / cos θ
cot θ = 1 / tan θ

Fundamental Identities
1. $\sin^2 \theta + \cos^2 \theta = 1$
2. $1 + \tan^2 \theta = \sec^2 \theta$
3. $1 + \cot^2 \theta = \text{cosec}^2 \theta$

Ratio	0^0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
Sin	0	1/2	1/√2	√3/2	1	0	-1	0
Cos	1	√3/2	1/√2	1/2	0	-1	0	1
Tan	0	1/√3	1	√3	Undefined	0	Undefined	0

Finding Co- sine and Sine Ratios

Angle 0^0	**30^0**	**45^0**	**60^0**	**90^0**	Steps to find Sine ratio
0	1	2	3	4	1. Write numbers 1 to 4
0	¼	1/2	3/4	1	2. Divide each number by 4
0	**1/2**	**1/√2**	**√3/2**	**1**	3. Take the square root
1	**√3/2**	**1/√2**	**1/2**	**0**	⇐ Cosine Ratios

Note: cos θ = sin (90 - θ). To find Cosine ratios, take numbers 4 to 0 and repeat steps 2 and 3.
tan θ = sin θ / cos θ

In Δ ABC, if a, b, and c denotes the sides opposite to angles A, B and C respectively. then

Sine Rule: In a given triangle (Δ ABC) the lengths of sides (a, b, c) of a triangle are proportional to the sine of their opposite angles ($\angle A$, $\angle B$, $\angle C$)

$$\mathbf{a / \sin A = b / \sin B = c / \sin C}$$

Cosine Rule: In a Δ ABC, :

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = c^2 + a^2 - 2ca \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Area of triangle ABC = (1/2) bc Sin A = (1/2) ac Sin B =(1/2) ab Sin C.

Trigonometrical ratios for sum and difference:

1. $\sin (A + B) = \sin A \cos B + \cos A \sin B$.

2. $\sin (A - B) = \sin A \cos B - \cos A \sin B$.

3. $\cos (A + B) = \cos A \cos B - \sin A \sin B$

4. $\cos (A - B) = \cos A \cos B + \sin A \sin B$

5. $\tan (A + B) = (\tan A + \tan B) / (1 - \tan A \tan B)$.

6. $\tan (A - B) = (\tan A - \tan B) / (1 + \tan A \tan B)$.

7. $\sin 2A = 2\sin A \cos A = 2\tan A / (1 + \tan^2 A)$

8. $\cos 2A = \cos^2 A - \sin^2 A = (1 - \tan^2 A) / (1 + \tan^2 A) = 1 - 2\sin^2 A = 2\cos^2 A - 1$.

9. $\tan 2A = 2 \tan A / (1 - \tan^2 A)$

10. $\sin 3A = 3 \sin A - 4 \sin^3 A$.

11. $\cos 3A = 4 \cos^3 A - 3 \cos A$.

12. $\tan 3A = (3 \tan A - \tan^3 A) / (1 - 3 \tan^2 A)$.

SOLVED EXAMPLES

Ex. Express the given degree measures in radians. a. 45^0 b. 150^0 c. 270^0

Sol. $x^0 = [(\pi / 180) \times x^0]^c$
$45^0 = (\pi / 180) \times 45 = \pi / 4$
$150^0 = (\pi / 180) \times 150 = 5\pi / 6$
$270^0 = (\pi / 180) \times 270 = 3\pi / 2$

Ex. Find the area of sector of a circle formed by an arc subtending an angle of 60 degrees at the centre. Also find the length of the arc if the diameter of the circle is 18 mm.

Sol. Length of the arc = s = $9 \times [60 \times (\pi / 180)] = 3 \pi$ mm.
Area of the corresponding sector = $^1/_2 \times r^2 \times \theta = {}^1/_2 \times 81 \times 60 \pi/180 = 81 \pi / 6$ mm^2.

Ex. Find the area of Δ PQR in which PQ = 4, QR = 15 and $\angle Q = 45^0$.

Sol. Area of Δ PQR = $^1/_2 \times 4 \times 15 \times \sin 45 = {}^1/_2 \times 60 \times {}^1/_{\sqrt{2}} = 30/\sqrt{2}$.

Ex. If $\cos\theta = \sqrt{3} / 5$ find $\sin\theta$ and $\tan\theta$.

Sol. $\cos\theta = \sqrt{3} / 5$, $\cos^2\theta = 3 / 25$; $\sin^2\theta = 1 - \cos^2\theta = 1 - (3/25) = 22/25$; $\sin\theta = \sqrt{(22/25)}$
$\tan\theta = \sin\theta / \cos\theta = \sqrt{(22/3)}$

Ex. If sin 20 is 0.3420. Find cos 70. Also find the value of (sin 15 / cos 75).

Sol. sin 20 = cos (90 - 70) = cos 70; cos 70 = 0.3420;
sin 15 = sin (90 - 75) = cos 75; (sin 15 / cos 75) = 1

Ex. In triangle ABC, a = 8, b = 4 and c = $4\sqrt{3}$. Find angle C.

Sol. From Cosine rule, we have, $\text{Cos } C = (a^2 + b^2 - c^2) / 2ab$
$\text{Cos } C = (64 + 16 - 48) / (2 \times 8 \times 4) = ½$; $C = \cos^{-1}(1/2) = 60^0$.

Ex. The difference between measures of two angles is $\pi/18$ and the sum of their measures is 420^0. Find the measure of angles in degrees.

Sol. $\pi / 18 = (\pi / 18) \times (180 / \pi) = 10^0$
Let the two angles be x and y. Following two equations can be formed from the given conditions. Solving these simultaneously will give the measure of two angles in degrees.

$$\begin{array}{rl} & x - y = 10 \\ + & x + y = 420 \\ \hline & 2x = 430;\ x = 215 \text{ and } y = 205 \end{array}$$

Ex. In Δ PQR, $\angle P = 53$, $\angle Q = 20$ and p = 2.7 m. Calculate r. (Given: cos 37 = 0.80, sin 20 = 0.34, sin 107 = 0.96)

Sol. $\angle P = 53$, $\angle Q = 20$ thus, $\angle R = 180 - (53 + 20) = 107$.
From the sin rule, r can be found in following manner.

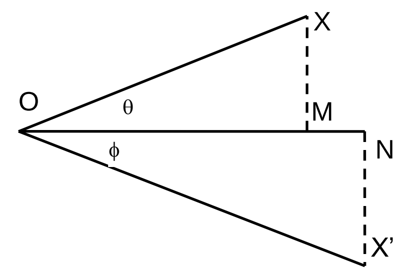
$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}$$

$\therefore r = (p/\sin P) \times \sin R = (2.7 / 0.80) \times 0.96 = 3.24$

HEIGHTS AND DISTANCES

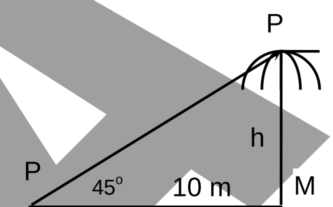
Angle of Elevation : Let point X be observed from point O. If point X is at higher level than O and if XM $\perp$ OM. Then $\theta = \angle MOX$ is called the angle of elevation.

Angle of Depression : If point X' is at a lower level than point O, then $\phi = \angle NOX'$ is called the angle of depression.



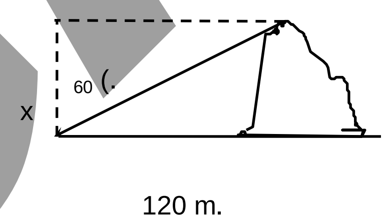
Ex. A person at a distance of 10 m from a tree finds the angle of elevation of the top of the tree to be 45^0. Find the height of the tree.

Sol. Let PM = h be the height of the tree.
tan 45 = h/OM = h/10
h = 10 $\times$ tan 45
= 10 $\times$ 1 = 10 m.



***Ex.*.** Angle of depression from top of a hill is sixty degrees. If the point on the ground is 120m away from the foot of the hill, find the height of the hill.

Sol. Let x denote the height of the hill as shown.
tan 30 = 120/x
$1/\sqrt{3} = 120/x$
$x = 120\sqrt{3}$



Ex. A swimmer, from one end of a swimming pool looks at the person on the diving board situated at the far end of the pool. He finds the angle of elevation to be thirty degrees. On swimming 10 m. towards the diving board, the angle of elevation is changed to forty-five degrees. Find the height of the diving board from the surface of the water.

Sol. Let the person swim from point O to point O'

$\angle POQ = 30^0$ tan 30 = h/(10 + x)

$1/\sqrt{3} = h/(10 + x) \ldots (1)$

$\angle PO'Q = 45^0$ tan 45 = h/x

1 = h/x

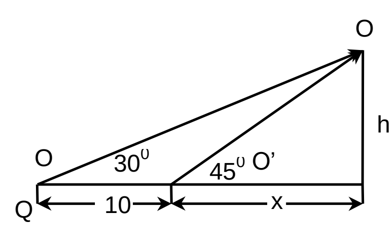
h = x

Substituting h = x in eqn.(1), $h/(10 + h) = 1/\sqrt{3}$

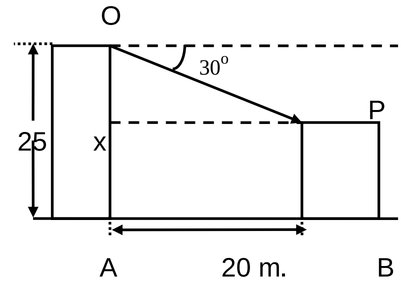
$\sqrt{3} \times h = 10 + h$

$h = 10/(\sqrt{3} - 1)$

h = 13.66 m.



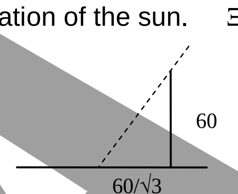
4. Angle of depression of the top of a building observed from another building 25 m. high is 30^0. If the distance between the two buildings is 20 m. Find height of the building.



tan 60 = PX / OX
i.e., $\sqrt{3}$ = 20 / OX
$\therefore$ OX = 20/$\sqrt{3}$

Height of the building = PB = AX
= 25 - (20/$\sqrt{3}$)

5. A tower 60m tall has a shadow 60/$\sqrt{3}$ m in length. Find the elevation of the sun. Ξ

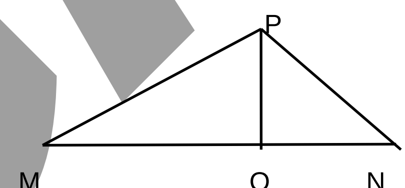


$\tan \theta = 60 / (60/\sqrt{3}) = \sqrt{3} \Rightarrow \theta = 60^o$

EXERCISES

1. From top of a building, 50 m high the angle of elevation of the top of a tower is found to be 30^0 and the angle of depression of the foot of the tower is also 30^0. Find height of the tower and its distance from the building.

2. A vertical pole 10 m tall is tied by two ropes as shown. in the adjoining figure. If the angle of elevation of top of the pole from M and N is 30^0 and 45^0 respectively, find the distance MN.

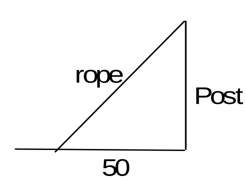


3. From a bank of a river, for a person the angle of elevation of the top of a tree situated on the opposite bank is 45^0. If the person steps back by 50 m the angle of elevation changes to 30^0. Find the width of the river.

4. Two sides of a valley meeting in a line at the bottom make angles 30^0 and 60^0 with the horizontal plane. A bridge of length 500$\sqrt{3}$ m spans across the valley. Find the height of the bridge above the bottom of the valley.

5. An aeroplane is flying with a uniform velocity at height of 100 m. At a particular instant, its angle of elevation is 45^0. After 10 seconds it is 30^0. Find the velocity of the aeroplane in m/s.

6. A 100 feet long rope is used to tie a post as shown. If the difference between the foot of the post and the rope is 50 feet, find the height of the post.



7. The length of each of the slanting sides of a sloping roof is 15 m. If the inclination of the roof is 30^0, and the eaves level is 12 m above ground level, find the level of the ridge from the ground (eaves is the lowest point of the sloping portion and ridge is the highest point of the roof).

8. The angle of elevation of a ship as seen from a submarine is 45^0. If the submarine is 200 m below sea-level, and if the ship is moving away from the submarine at the speed of 20 ($\sqrt{3}$ - 1) m/s, at what angle should a missile be fired from the submarine so as to hit the ship ? The missile travels at the speed of 40 m/s. (N.B. : the lines of travel of the ship and the missile lie in the same vertical plane).

9. The angle of depression of a boat as observed form a cliff is 60^0, and the angle of elevation of an aircraft from the same point is 30^0. If the altitude of the aircraft is 500 m above sea level, and the cliff is 100 m high, find the angle of elevation as seen from the boat.

10. From the equator, a man travels due north up to a distance of one-sixth the equator. Find the ratio of the latitude at this point to the equator.

11. Prove that $\sin^6 A + \cos^6 A + 3 \sin^2 A \cos^2 A = 1$.

12. Prove that (cot A + tan B) / (tan A + cot B) = cot A tan B.

13. Prove that (sin A + cos A).(tan A + cot A) = cosec A + sec A..

14. What is the value of $(1 - \tan^2 15°) / (1 + \tan^2 15°)$?

15. If tan x = 1/2 and tan y = 1/3, show that $x + y = \pi/4$.

16. The angle of elevation of the top of a pillar at any point A on the ground is 15^0 . On walking 100m towards the pillar, the angle becomes 30^0. Find the height of the pillar and its distance from point A.

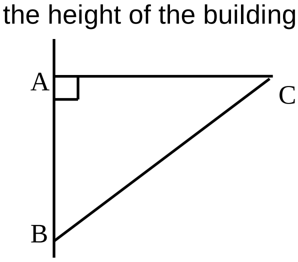
17. The shadow of a tower standing on a level ground is found to be 60 metres longer when the sun's altitude is 30^0 than when it is 45^0. Find the height of the tower.

18. A man in a boat rowed away from a cliff 150 metres high takes 2 minutes to change the angle of elevation of the top of the cliff from 60^0 to 45^0 . Find the speed of the boat.

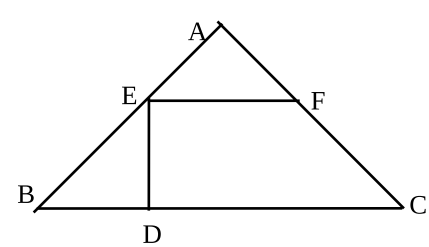
19. A tower 51m high has a mark at a height of 25m from the ground. Find at what distance the two parts subtend equal angles to an eye at the height of 5m from the ground.

20. A chimney 20 metres high, standing on the top of a building subtends an angle whose tangent is 1/6 at a distance 70 metres from the foot of the building. Find the height of the building.

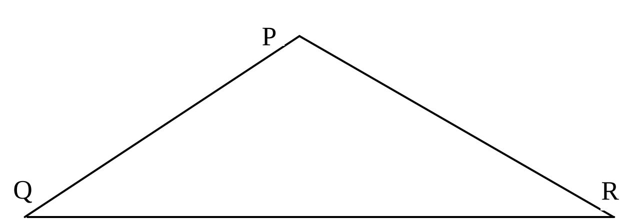
21. A ship leaves on a long voyage, from the point B on the sea - shore, at 7:00 a.m. The ship travels at the speed of 28 ¼ kmph in the direction BC. Another ship, travelling at the speed of 56 kmph, leaves the sea - shore from the point A at 9:00 a.m. on the same day, to deliver some mail at the point C. If the two ships meet at the point C at 11:0 a.m., what is the distance AB?



22. In the adjoining figure, ED ⊥ BC, ED ⊥ EF. If l(ED) = 40 units, l(BD) = 9 units, l(AE) = 82 units, l(AF) = 90 units. Find l(FC).



23. In the figure, l(PQ) = 3 units, l(PR) = $\sqrt{13}$ units and l(QR) = 4 units. Find m∠ PQR.



24. An aeroplane flying horizontally, √3 km above the ground is observed at an angle of elevation of 45^0. If after 10√3 (√3 - 1) seconds the elevation is observed at 30^0 , what is the speed of the aeroplane (in kmph)?

ANSWERS

1. 100 m.50√3 m.
2. 10 (1 + √3) m.
3. 68.30 or, 50 / (√3 - 1)
4. 375 m.

5. 7.32 m/s
6. 50 √3
7. 19.5 m
8. 30^0

9. $\tan^{-1}$ (5√3 / 11)
10..1 : 2.
14. √3/2.

16. 50, 50(2+√3).
17. 30 (√3 + 1)
18. 25 (3 - √3)
19. 160

20. 50.
21. 15 km.
22. 45 units.
23. 60º.

24. 360 kmph.

Concepts 6

Mensuration

Units of Measurement :

Length : **For any measure :**

10 millimetres	=	1 cm	**mili = 10^{-3}**
10 centimetres	=	1 dm	**centi = 10^{-2}**
10 decimetres	=	1 m	**deci = 10^{-1}**
10 metres	=	1 Dm	**deca = 10 .**
10 decametres	=	1 hm	**hecto = 10^{2}**
10 hectometres	=	1 km (kilometre)	**kilo = 10^{3}**

Area is expressed in square units, volume is expressed in cubic units.

1 acre = 4047 sq. m., 1 hectare = 10000 sq. m. 1 litre = 1000 cc.

Weight = volume x density

Areas
Area of a plane figure is the surface enclosed by its sides.

Triangle :
Area = $^1/_2$ base x height

Area = $\sqrt{[s(s - a)(s - b)(s - c)]}$ where s = semi perimeter = (a + b + c)/2

Area = $^{\sqrt{3}}/_4$ side^2 for an equilateral triangle

Area = $^1/_2$ product of perpendicular sides for a right triangle

Rectangle :
Area = length x breadth

Square :
Area = side^2 = a^2

Diagonal = $a\sqrt{2}$

Area = $^1/_2$ product of diagonals

Parallelogram :
Area = base x height

Rhombus :
Area = $^1/_2$ product of diagonals

Trapezium :
Area = $^1/_2$ (sum of parallel sides) x distance between parallel sides = $^1/_2$ (a + b)h

Circle :
Area : πr^2 where r = radius of the circle; Circumference = $2\pi r$

Area of a sector of a circle subtending an angle θ at the centre of the circle is $r^2 . \theta/2$, where θ is in radians.

Area of circular ring = $\pi (r_1^2 - r_2^2) = \pi (r_1 + r_2)(r_1 - r_2)$ where r_1 = external radius, r_2 = internal radius

Areas and volumes of Cuboids & Cubes :
Any thing which occupies space is called a solid. It has three dimensions length, breadth & height.

A **cuboid** is a rectangular solid having six faces all of which are rectangles.

A rectangular solid whose every face is a square is called a **cube.**

Cuboid :
Volume **=** l x b x h where l = length, b = breadth, h = height

Area of 4 walls of a cuboid = 2(l + b) x h

Total surface area of a cuboid = 2(lb + bh + lh)

Body Diagonals of a cuboid = $\sqrt{(l^2 + b^2 + h^2)}$

Cube :
Volume = a^3 where a = edge of the cube

Total surface area of the cube = $6a^2$, Body Diagonal of a cube = $a\sqrt{3}$

- Volume of material = external volume - internal volume
- If l, b, h are the external dimensions of a closed cuboid of thickness x, then internal dimensions are (l - 2x), (b - 2x), (h - 2x).
- If the cuboid is an open cuboid then internal dimensions = (l - 2x), (b - 2x), (h - x)

Cylinder :
If a rectangle is revolved about its one side as its axis, the solid thus formed is called a right circular cylinder.

Volume = $\pi r^2 h$ where r = radius of base, h = height

Curved surface = $2\pi rh$

Total surface = $2\pi r(r + h)$

Volume of material in a hollow pipe = $\pi (R^2 - r^2)L$ where R = external radius, r = internal radius, L = length of pipe

Total surface of an open pipe = $2\pi[Rh + rh + (R^2 - r^2)]$

Cone :
If a right triangle is revolved about one of its sides containing the right angle, the solid formed is called a cone.

Volume = $^1/_3 \pi r^2 h$ where r = radius of circular base, h = height

Curved surface area = $\pi r l$ where l = slant height = $\sqrt{(r^2 + h^2)}$

Total surface area $= \pi r(r + l)$

Sphere :
When a circle is revolved about its diameter, the solid thus formed is called a sphere.
Volume $= {}^4/_3\, \pi r^3$ where r = radius of the sphere

Surface $= 4\pi r^2$

Volume of a spherical shell $= {}^4/_3\, \pi (R^3 - r^3)$

Volume of a hemisphere is ${}^2/_3\, \pi r^3$ and its total surface area is $3\pi r^2$

Summary of Formulae

Areas of different plane figures, Surface areas of solids :

Plane/Solid	Area	Remarks
Triangle (any)	${}^1/_2\, b h$	**b = base, h = height**
Right angled triangle	${}^1/_2\, b h$	**b = base, h = height**
Equilateral triangle	${}^{\sqrt{3}}/_4\, a^2$	**a = any side**
Isosceles triangle	${}^b/_4 \sqrt{(4a^2 - b^2)}$	**b = base, a = any of the two equal sides**
Quadrilateral (any)	${}^1/_2\, d (p_1 + p_2)$	**d = diagonal, p_1, p_2 perpendiculars to diagonals from opposite vertices**
Square	a^2	**a = any side**
Rectangle	$l\, b$	**l = length. b = breadth**
Parallelogram	$b\, h$	**b = base, h = height**
Rhombus	${}^1/_2 d_1 d_2$	**d_1, d_2 = diagonals**
Trapezium	${}^1/_2\, h (s_1 + s_2)$	**s_1, s_2 = parallel sides, h = height**
Kite	${}^1/_2 d_1 d_2$	**d_1, d_2 = diagonals**
Regular Hexagon	${}^{3\sqrt{3}}/_2\, a^2$	**a = any side**
Circle	πr^2	**r = radius**
Cube	$6a^2$	**a = any edge**
Circular cylinder	$\pi r^2 h$	**r = radius, h = height**

Volumes of different solids

Solid	Volume	Remarks
Prism	$b\, h$	**b = base area, h = height**
Cube	a^3	**a = any side**
Rectangular box	$l\, b\, h$	**l = length, b = breadth, h = height**
Sphere	${}^4/_3\, \pi r^3$	**r = radius**
Cone	${}^1/_3\, \pi r^2 h$	**r = radius of base, h = height**
Circular cylinder	$\pi r^2 h$	**r = radius of base, h = height**

Total Surface area of different solids

Solid	Surface Area	Remarks
Prism	$p\, h + 2\, b$	**p = perimeter of base, h = height, b = base area**
Rectangular box	$2l\, h + 2\, l\, b + 2\, b\, h$	**l = length. b = breadth, h = height**
Circular cylinder	$2\, \pi r (r + h)$	**r = radius of base, h = height**
Cube	$6a^2$	**a = any edge**
Cone	$\pi r(r + l)$	**r = radius of base, l = slant height**
Pyramid	$b + {}^1/_2\, p\, l$	**b = base area, p = perimeter of base, l = slant height**
Sphere	$4\, \pi r^2$	**r = radius**

Solved Examples:

1. The area of a playground is 5600 sq. metres. What will be the cost of covering it with gravel 1 cm. deep if the cost of gravel is Rs. 2.80 per cubic metre ?

Ans. Volume of gravel = (5600) x ($^{1}/_{100}$) = 56 cubic metres.
Cost of gravel = Rs. (56 x 2.80) = Rs. 156.80

2. There is a rectangular plot of area 43560 sq. ft. The length and breadth of the rectangle are in the ratio 5 : 2. A gravel path 5 ft. wide runs outside the plot close to the four sides of the plot. If the cost of gravelling is Rs. 590 at 25 paise per cubic feet, what is the depth to which the gravelling has been done ?

Ans. Volume of the gravel path = (total cost/rate) = 59000/25 = 2360 cu. ft.

Let the length and breadth of the plot be 5x and 2x feet respectively
∴ the area = $10x^2$ sq. ft.
$10x^2 = 43560$
∴ x = 66
∴ the length = 330 ft. breadth = 132 ft.
Area of the plot including the path = (330 + 10) (132 + 10) = 48280 sq. ft.
∴ Area of the path = (48280 - 43560) = 4720 sq. ft.
Depth of the path = Volume / Area = $^{2360}/_{4720}$ ft = $^{1}/_{2}$ ft. = 6 inches.

3. There is a rectangular water tank in which water stands to a depth of 6.5 metres. The base of the tank is 80 metres by 60 metres. The tank has connected to it an outlet pipe of square cross section of side 20 cm. In what time will the tank be emptied if water runs through the pipe at the rate of 15 km. per hour. ?

Ans. Volume of water present = (80 x 60 x 6.5) = 31200 cu. m.
Area of cross section of the pipe = ($^{20}/_{100}$ x $^{20}/_{100}$) sq. m. = $^{1}/_{25}$ sq. m.
Volume of water that will be emptied within an hour = ($^{15 \times 1000 \times 1}/_{25}$) = 600 cu. m.
Time taken to empty the reservoir = $^{31200}/_{600}$ = 52 hours.

4. An open rectangular tank made of iron was measured from outside and the following measurements were recorded :
Length = 1 m. 35 cm. Breadth = 1 m. 8. cm. Height = 90 cm. Thickness = 2.5 cm
What is the capacity of the tank and what is the volume of the iron used ?

Ans. External dimensions: l = 135 cm, b = 108 cm, d = 90 cm.
Internal dimensions: l = 130 cm. b = 103 cm, d = 87.5 cm.
∴ Capacity = Internal volume = (130 x 103 x 87.5) = 1171625 cu. cm
Volume of iron used = (outer volume) - (inner volume)
= (135 x 108 x 90) - (1171625)
= 140575 cu. cm.

5. Cost of painting four walls of a room 45 metres x 20 metres at Rs. 5 per sq. m. is Rs. 6500. Find the height of the room.

Ans. Total area to be painted = (6500/5) = 1300 sq. m.
If the height is h, the area of the walls 2 x (45 x h + 20 x h) = 1300
∴ h = 10 m

6. The lower part of a tent is a right circular cylinder and its upper part is a right cone. The diameter of the base is 70 m and the total height is 15 m and the height of the cylindrical part is 3 m. Find the cost of material at Rs. 10 per sq. m

Ans. Area of material required = Curved surface area of the cone + curved surface area of the cylinder = $\pi r l + 2\pi r h$.

$l = 37$ (r = 35, h = 12 triplet of Pythagoras)
$\therefore$ Area of material = $\pi \times 35 \times 37 + 2 \times \pi \times 35 \times 3 = 4730$ sq. m.
cost = Rs. 4730 x 10 = Rs. 47300

7. What is the number of edges of a solid having 10 faces and 16 vertices. ?

Ans. F + V = E + 2 (Euler's Theorem)
10 + 16 = E + 2 $\therefore$ number of edges = 24

8. The radius of a right circular cylinder is increased by 50%. What is the increase in volume ?

Ans. Earlier radius = r, new radius = 3r/2
New Volume/Old Volume = $(\pi \times 9r^2/4 \times h)/(\pi r^2 h) = 9/4$
If the old volume is 4, new volume = 9 Increase = 5 i.e. 125%

9. The front wheel of a cart is 2π feet and back wheel is 3π feet in circumference. Find the distance travelled when the front wheel has made 10 revolutions more than the hind wheel.

Ans. If the back wheel makes n revolutions, distance tarvelled = $n \times 3\pi = (n + 10)\, 2\pi$
$\therefore$ n = 20
$\therefore$ distance travelled = $20 \times 3\pi = 60\pi$

10. There is a prism with a triangular base. the sides of the base are 17 cm., 25 cm. and 28 cm. The volume of the prism is 4200 sq. cm. What is the height of the prism?

Ans. Volume of a prism = Area of the base x height
a = 17 cm. b = 25 cm c = 28 cm.
$S = {}^{(17 + 25 + 28)}/_{2} = 35$ cm.
Area of base = $\sqrt{[\, S(S - a)(S - b)(S - c)\,]}$
= $\sqrt{[35(35 - 17)(35 - 25)(35 - 28)\,]}$
= 210 sq. cm
Height of the prism = Volume / Area of base = 4200 / 210 = 20 cm

11. A right pyramid has a rectangular base. The sides of which are 24 cm. and 16 cm. If each of the slant edges is 17 cm., what is the volume of the pyramid?

Ans. Height of the pyramid = $\sqrt{[\,(17)^2 - (12^2 + 8^2)]} = 9$ cm.
Volume = ${}^{1}/_{3}$ x Area of base x height
= ${}^{1}/_{3} \times 24 \times 16 \times 9$
= 1152 cu. cm.

12. In a foundry a solid rectangular block of iron is cast into a pipe. The dimensions of the block are 4.4 m., 2.6 m. and 1 m. The pipe has a internal diameter of 60 cm and external diameter of 70 cm. What will be the length of the pipe ?

Ans. Volume of the block = (440 x 260 x 100) cu. cm.
Internal radius of the pipe = 30 cm. External radius of the pipe = 35 cm.
Let the length of the pipe = h
External Volume = $\pi \times 35 \times 35 \times h$ Internal volume = $\pi \times 30 \times 30 \times h$
Volume of pipe = External Volume - Internal volume = $(3850 - {}^{19800}/_{7})\, h = {}^{7150}/_{7} h$
${}^{7150}/_{7}\, h = 440 \times 260 \times 100$ $\therefore$ h = 112 metres

13. A well with 11.2 metres inside diameter has a depth of 8 m. Earth taken out of the well is spread evenly all round it to a width of 7 m to form an embankment. What is the height of the embankment?

Ans. The embankment is a circular path 7 m wide around the well. Radius of the well = 5.6 m.
Radius of the well with embankment = 5.6 + 7 = 12.6 m
Volume of the earth taken out = $\pi \times 5.6 \times 5.6 \times 8 = 788.48$ cu. m.
Area of the embankment = $(\pi \times 12.6 \times 12.6) - (\pi \times 5.6 \times 5.6) = 400.4$ sq. m.
Height of the embankment = ${}^{Volume}/_{Area} = {}^{788.48}/_{400.4} = 1.97$ metres

14. A right circular cylinder of height 10 cm. and radius of base 6 cm. is taken. From this cylinder a right circular cone of the same height and base is removed. What will be the volume of the remaining solid ?

Ans. Volume of the cylinder = $\pi \times 6 \times 6 \times 10 = 360\pi$
Volume of the cone = ${}^{1}/_{3} \times \pi \times 6 \times 6 \times 10 = 120\pi$
Volume of the remaining solid = 240π

15. There is a conical vessel with internal radius 10 cm. and height 48 cm. this vessel is full of water. This water is then poured into a cylindrical vessel of internal radius 20 cm. What will be the height to which the water will rise in this cylinder?

Ans. Volume of Water = ${}^{1}/_{3} \times \pi \times 10 \times 10 \times 48 = 1600\pi$
Let the height to which the water rises be h cm.
Volume of water in the cylindrical vessel = $\pi \times 20 \times 20 \times h = 400\pi h$
But $400\pi h = 1600\pi$
$\therefore$ h = 4 cm.

16. A right-angled triangle with perpendicular sides 6.3 cm. and 10cm. is made to turn round on the longer side. What is the volume and surface area of the cone thus formed?

Ans. The height of the cone formed is 10 cm. and the radius of the base is 6.3 cm.
$\therefore$ the volume of the cone = ${}^{1}/_{3} \times \pi \times 6.3 \times 6.3 \times 10 = 415.8$ cu. cm.
Slant height = $\sqrt{(10)^2 + (6.3)^2} = 11.82$ cm.
Surface area = $\pi \times 6.3 \times (11.82 + 6.3)$ sq. cm. = 358.776 sq. cm.

17. A hemisphere of lead of radius 7 cm. is cast into a right circular cone of height 49 cm. What is the radius of the base?

Ans. Volume of the hemisphere = ${}^{1}/_{2} \times {}^{4}/_{3} \times \pi \times 7 \times 7 \times 7 = {}^{2156}/_{3}$
Let the radius of the base of the cone be r cm.
${}^{1}/_{3} \times \pi \times r^2 \times 49 = {}^{2156}/_{3}$
r = $\sqrt{14}$

18. A solid is made up of a cylinder with hemispherical ends. If the whole length of the solid is 9 metres and its diameter is 3 metres what will be the cost of painting its surface at Rs. 7 per sq. m ?

Ans. Length of the cylindrical portion = (9 - 3) = 6 m.
Radius of the cylinder = ${}^{3}/_{2}$ metres
Curved surface of cylindrical portion = $(2 \times \pi \times {}^{3}/_{2} \times 6) = {}^{396}/_{7}$ sq. metres.
Curved surface of the two hemispheres each of radius ${}^{3}/_{2}$ metres
= $2 \times (2 \times \pi \times {}^{3}/_{2} \times {}^{3}/_{2}) = {}^{198}/_{7}$ metres
Total Area to be painted = $({}^{396}/_{7} + {}^{198}/_{7}) = {}^{594}/_{7}$ metres
Cost of polishing = ${}^{594}/_{7} \times 7$ = Rs. 594

19. A metal sphere of diameter 42 cm. is dropped into a cylindrical vessel partly filled with water. The diameter of the vessel is 1.68 metres. If the sphere is completely submerged, find by how much will the surface of water rise?

Ans. Radius of the sphere = 21 cm.
Volume of the sphere = ($^4/_3$ x π x 21 x 21 x 21) cu. cm. = 38808 cu. cm.
Volume of water displaced by the sphere = 38808 cu. cm.
Let the water rise by h cm.
π x 84 x 84 x h = 38808x h = 1.75 cm.

Exercises

1. A rectangular tank 25 cm long and 20 cm wide contains water to the depth of 5 cm. A metal cube of side 8 cm is placed in the tank so that one face of the cube rests at the bottom. Find out how much water must be poured into the tank so that the cube is covered.

2. The rain water from a flat roof 5m by 7 m drains into a tank with dimensions 42 cm, 20 cm and 50 cm. What depth of the rainfall will fill the tank?

3. A swimming pool 25 ft. by 10 ft. is 4 ft. deep along 8 ft of the length. It gradually slopes to 10 ft. depth in the remaining 17 ft. What is the volume of water that the pool can hold? What is the depth of the pool at 16.5 ft. from the shallow end?

4. A part of swimming pool is 15 ft. by 10 ft. and is 4 ft. deep from the shallow end up to 9 ft. till the deep end and the other part is semicircular with a constant depth of 9 ft. What volume of water will it hold?

5. The thickness of a hollow cylinder is 3.5 cm and its outside diameter is 35 cm. Find the cost of painting its surface at the rate of 5 paise/sq. cm if the cylinder is 70 cm long?

6. Find the weight of a lead pipe 3.5 m long if the external diameter of the pipe is 2.4 cm and the thickness of lead is 2 mm and 1 cc of lead weighs 11.4 gm.

7. The volume of a sphere is numerically equal to its surface area. Find its diameter.

8. A spherical shell of metal has outer radius of 9 cm and inner radius of 8 cm. If the metal costs Rs. 1.80 per cc, what is the cost of the shell?

9. A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. The diameters of two of these are 1.5 cm and 2 cm respectively. What is the diameter of the third?

10. A hollow cone 24 cm high with base radius 7 cm is required to be made. What is the area of sheet metal required including the base?

11. The cost of silver plating the interior of a hemispherical bowl of diameter 14 cm is Rs. 215.60. If the cost per unit area of silver plating remains the same what will it cost to silver plate a similar bowl of 16 cm diameter?

12. One cubic cm of gold is drawn into a wire of diameter 0.1 mm. What will be the length of this wire?

13. A cone of height 7 cm. and base radius 3 cm is carved out of a rectangular block of wood 10 cm x 5 cm x 2 cm. What is the percentage of the wood wasted?

14. How many spherical bullets can be made out of a cube of lead whose edge measures 22 cm. each bullet being 2 cm. in diameter?

15. If one cubic cm. of iron weighs 21 gm. then what will be the weight of a cast iron pipe of length 1 metre with a bore of 3 cm. in which the thickness of metal is 1 cm?

16. A sculptor needs 12 litres of paint to paint a piece, cubical in shape. If every side of the cube is increased by 50 percent, how much extra paint would the sculptor need to paint the cube now? Assuming he paints each surface uniformly.

17. The floor of a swimming pool is to be covered using tiles of dimensions 8cm $\times$ 12 cm. The length and the breadth of the pool is 48 and 24 meters respectively. The depth of the pool at the shallow end is 1 m and at the deep end is 21 m. How many pieces of tiles are required to cover the floor of the pool completely?

18. What is the number of steel balls of radius 2 mm that can be produced by melting a bigger steel ball of radius 10cm?

19. The contents of a cylindrical cask are emptied in a conical vessel. The cask is 21 cm in height and its diameter is 6 cm. What is the height up to which the liquid level rises in the conical vessel, if its diameter at the base is 9 cm?

20. What is the volume of a box made from cutting 7 X 7 cm squares from the four corner of a 21 X 28 cm cardboard rectangle and folding the piece along the cuts?

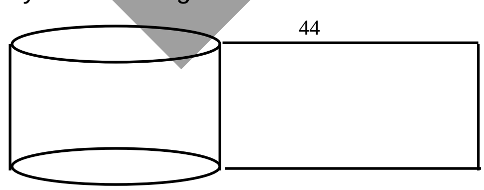
21. What is the size of the longest pencil that can fit in a box, with dimensions 12 cmX10cmX8 cm?

22. A metallic cube of side d is melted and is cast back into a cone and a cylinder. The heights and the radii of the cylinder and the cone are d/2. What is the percentage of unused liquid metal ?

23. A pond is to be constructed by digging out 12 x 10 x 8 m of earth. The soil dug out is to be used in filling up 3 smaller ponds of dimensions 10 x 6 x 4m. If the remaining soil is used in a piece of land 10 x 8 m, what is the height of the layer of soil on the ground?

24. A golden sphere is melted, and is drawn into a thin wire. What is the length of the wire drawn, if the radii of the sphere and the wire are 3 cm and 2 mm respectively?

25. A rectangular sheet of paper, 44 cm by 10 cm can be exactly wrapped around the curved surface of a cylinder of height 10 cm. Find the volume of the cylinder?



26. A slab of iron 11 inches in length, 10 inches in breadth and 2 inches thick was melted and resolidified in to the form of a solid circular cone of radius $\sqrt{21}$ inches. Find the height of the cone formed?

27. A copper wire when bent in the form of a equilateral triangle of area $16\sqrt{3}$ cm^2. If the same wire is bent into the form of a regular hexagon, find the area of the regular hexagon?

28. Circles of radii $\sqrt{7}$ are cut out from each face of a cube of side $4\sqrt{11}$ cm. What is the ratio of the remaining area to the area cut out from the cube?

29. The radii of four spheres are in the ratio of 1: 9: 10: 12. Select any two spheres such that sum of volumes of two selected spheres is equal to that of unselected spheres.

30. A cylindrical cistern of radius 8 cm. is partly filled with water. Three solid spherical balls of radius 4 cm are completely immersed in the water. By how much will the water level in the cistern rise?

ANSWERS

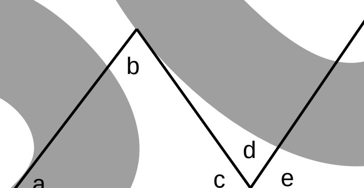
1. 988 cm^3	**2.** 1.2 mm	**3.** 1510 ft^3 & 7ft	**4.** $975+(225/2)\pi$ ft^3	**5.** Rs. 727.65
6. 5517.6 gm	**7.** 6 units	**8.** Rs. 1636.80	**9.** 5/2 cm	**10**. 704 cm^2
11. Rs. 281.60	**12**. 127.30 m.	**13.** 34%	**14**. 2541 bullets	**15**. 26.4 kg.
16. 15 litres	**17**. 130000	**18.** 125000	**19**. 28 cm	**20**. 686 cc
21. 17.5cm	**22**. 47.64	**23.** 3 m.	**24**. 9 m.	**25.** 1540 cm^2.
26. 10 inches.	**27.** $24\sqrt{3}$ cm^2.	**28.** 7 : 1.	**29.** 1^{st} & 4^{th} or 2^{nd} & 3^{rd}.	
30.4 cm.				

Revision of Rules

1. Two lines are said to be parallel only when their points of intersection is/are

   a. one point b. line c. ϕ d. none of these

2. In the figure given below, the sum of angle ‘a’ and angle ‘b’ is equal to



   a. $\angle c + \angle d$ b. $\angle d + \angle e$ c. $\angle b + \angle c$ d. $\angle a + \angle c$

3. In a triangle, interior opposite angle is always less than

   a. any one angle of the triangle b. the opposite angle
   c. a right angle d. the exterior angle

4. Sum of the two interior opposite angles of a triangle is always equal to

   a. exterior angle b. right angle c. 3^{rd} angle d. none of these

5. Sum of all the interior angles of a pentagon is equal to

   a. 360^o b. 540^o c. 820^o d. none of these

6. In a triangle the sum of the two angles is equal to the third angle, considering interior angles only, then the triangle is

   a. right angled b. acute angled c. equilateral d. none of these

7. Sum of the interior angles of a polygon having 'n' sides is equal to

a. $(n + 1)180^o$ b. $(2n - 4) 90^o$ c. $(2n + 4)90^o$ d. $(n + 1)360^o$

8. Two sides of a triangle are unequal. The angle just opposite to the larger side is

a. obtuse b. 180^o
c. greater than the angle opposite the smaller side d. none of these

9. The angle made by the altitude of a triangle with the side on which it is drawn is equal to

a. 90^o b. 60^o c. 30^o d. none of these

10. One angle of a triangle is greater than the other. The side opposite to it is

a. greater than the side opposite to the other
b. less than the side opposite to the other
c. equal to unity
d. none of the above

11. Sum of squares on the two perpendicular sides of a right angled triangle is equal to the square on the

a. base b. perpendicular c. hypotenuse d. none of these

12. In a parallelogram the opposite angles are

a. unequal b. equal c. complementary d. less than 90^o

13. A regular hexagon has been inscribed in a circle. the area of the hexagon will be

a. greater than the area of the circle b. less than the area of the circle
c. equal to the area of the circle d. cannot say

14. When the bisector of any angle is perpendicular to the opposite side, then the triangle is

a. isosceles b. equilateral c. right angled d. none of these

15. If two parallel lines are intersected by a transversal, then the bisectors of the interior angles so formed make a

a. rectangle b. square c. trapezium d. none of these

16. Each angle of a complementary set of angles must be

a. obtuse b. acute c. reflex d. none of these

17. Number of pairs of vertical angles formed when two lines intersect is/are

a. one b. two c. three d. four

18. If the bisectors of two adjacent angles are perpendicular, the adjacent angles are the angles of
a. linear pair b. collinear pair c. coplanar pair d. cannot say

19. The triangle formed by joining the midpoints of the sides of an equilateral triangle is

a. right angled b. obtuse angled c. scalene d. equilateral

20. The bisector of the angle at the vertex of an isosceles triangle

a. intersects the base when produced
b. bisects the base
c. bisects the base but is not perpendicular to it
d. bisects the base and is perpendicular to it

21. If two angles of a triangle are congruent, the sides opposite of these angles are

a. not equal b. congruent c. may be congruent d. cannot say

22. If the bisector of any angle of a triangle bisects its opposite side, the triangle is

a. equilateral b. scalene c. acute angled d. isosceles

23. Which of the following is a correct postulate of congruence of two triangles ?

a. SAS b. ASS c. SSA d. none of these

24. The straight line joining the midpoints of any two sides of a triangle is ______ to the third side

a. perpendicular b. parallel c. equal d. unequal

25. If the bisector of the vertical angle bisects the base, the triangle is
a. equilateral b. isosceles c. scalene d. none of these

26. The point of intersection of the medians of the triangle is called its
a. centroid b. incentre c. excentre d. orthocentre

27. The point of intersection of the altitudes of the triangle is called its
a. centroid b. incentre c. excentre d. orthocentre

28. The point of intersection of the angle bisectors of a triangle is called its
a. incentre b. centroid c. excentre d. orthocentre

29. In a triangle ABC, if the median BE is equal to the median CF, then the triangle is
a. equilateral b. isosceles c. right-angled d. none of these

30. In a triangle ABC, if altitude BE is equal to altitude CF, then the triangle is
a. equilateral b. right angled c. isosceles d. none of these

31. The angle between the internal bisector of one base angle and the external bisector of the other base angle is equal to
a. the vertical angle of the triangle
b. One-half the vertical angle
c. one-fourth the vertical angle
d. none of these

32. If three altitudes of a triangle are equal, the triangle is
a. isosceles b. equilateral c. right-angled d. none of these

33. The bisector of the exterior angle at the vertex of an isosceles triangle is
a. perpendicular to the base b. parallel to the base
c. bisector of the base d. none of these

34. The straight line drawn from the midpoint of a side of a triangle, parallel to the base is one that
a. bisects the other side b. intersects the other side at right angles
c. trisects the other side d. none of these

35. The median on the hypotenuse of a right angled triangle is equal to
a. the hypotenuse b. 1/3 the hypotenuse
c. 1/2 the hypotenuse d. Nothing can be said.

36. In an isosceles triangle ABC, D, E, F are the midpoints of the base BC and the equal sides AB, AC respectively, then
a. DC = BC b. DF = EF c. DF = DE d. DC= DE

37. Medians of a triangle pass through the same point which divides each median in the ratio
a. 1 : 3 b. 1 : 2 c. 2 : 1 d. 1 : 4

38. The sum of two medians of triangle is
a. less than the third b. greater than the third
c. equal to the third d. None of these

39. A median divides a triangle into two triangles of
a. equal area b. unequal area
c. areas in the ratio 2 : 1 d. none of these

40. A triangle can have at most one __________________ angle
a. acute b. obtuse c. straight d. none of these

41. If the diagonal of a quadrilateral bisect each other and are perpendicular, the quadrilateral is,
a. rectangle b. square c. rhombus d. none of these

42. The bisectors of a pair of opposite angles of a parallelogram are
a. perpendicular to each other b. parallel to each other
c. intersecting at a point d. none of these

43. If diagonal AC = diagonal BD and AC is perpendicular to BD in a parallelogram ABCD then it is
a. rectangle b. rhombus c. square d. none of these

44. Area of a rectangle and area of a parallelogram standing on the same base and between the same parallels have relation between them as
a. area of the parallelogram = $^{1}/_{2}$ the area of the rectangle
b. area of rectangle = $^{1}/_{2}$ the area of the parallelogram
c. there is no relation
d. they are equal

45. If the midpoints of the sides of a quadrilateral are joined, then the figure formed is a
a. quadrilateral b. triangle c. parallelogram d. none of these

46. If the diagonals of a parallelogram are equal then it is a
a. square b. rhombus c. rectangle d. none of these

47. Area of a triangle has the following relation with the area of a parallelogram when both of them are standing on the same base and between the same parallels
a. area of the triangle = area of the parallelogram
b. area of the triangle = twice the area of the parallelogram
c. area of the triangle = thrice the area of the parallelogram
d. none of these

48. A quadrilateral is a parallelogram if
a. a pair of opposite sides is equal
b. a pair of opposite sides is parallel
c. a pair of opposite sides is equal and parallel
d. none of these

49. A diagonal of a parallelogram divides it into
a. three triangles of equal area b. Two triangles of equal area
c. Four triangles of equal area d. none of these

50. Opposite angles of a parallelogram are
a. unequal b. equal c. always right angles d. none of these

51. In a triangle ABC , the median AD bisecting the side BC has its midpoint O. The line CO meets AB at E. AE is equal to
a. AB/2 b. 2AB/3 c. AB/3 d. AB/4

52. If a line is drawn parallel to one side of a triangle, the other two sides are divided
a. in the same ratio b. in the inverse ratio
c. in no equal ratio d. none of these

53. If the diagonals of a parallelogram are equal, it is a
a. rhombus b. quadrilateral c. rectangle d. none of these

54. Two triangles are similar if and only if
a. all the sides are equal
b. corresponding sides are proportional
c. corresponding angles are not proportional
d. none of these

55. AAA theorem is applicable for two triangles to prove them
a. congruent b. equilateral c. similar d. isosceles

56. The ratio of areas of similar triangles is equal to the ratio of
a. the sides b. the altitudes
c. squares on the corresponding sides d. none of these

57. If two chords of a circle intersect inside or outside a circle, the rectangle contained by the parts of one chord is equal in area to the rectangle contained by
a. the parts of the other b. circumference and the radius
c. the square of the diameter d. none of these

58. If the perpendicular drawn from the vertex of a right angled triangle to the hypotenuse, the number of similar triangles formed is equal to
a. 2 b. 4 c. 3 d. none of these

59. In triangle ABC, AD is perpendicular to BC. If AD^2 = BD.DC, the triangle is
a. obtuse angled b. acute angled
c. right angled d. none of these

60. In a parallelogram ABCD, E is a point on AD. AC and BE intersect each other at F. Then
a. BF.EF = FC.FA b. BF.FC = EF.FA
c. BF.FA = EF.FC d. none of these

61. P and Q are two points on the sides CA and CB of a triangle ABC, right angled at C. Then $AQ^2 + BP^2$ is equal to

a. $AB^2 + PC^2$ b. $AC^2 + PQ^2$
c. $AB^2 + PQ^2$ d. $BQ^2 + PQ^2$

62. Equal chords of a circle subtend equal angles at the

a. radius b. centre c. diameter d. none of these

63. The number of circles passing through three non collinear points is

a. two b. three c. one d. four

64. If two circles C1 and C2 have three points in common, then

a. C1 and C2 are concentric b. C1 and C2 are the same circle
c. C1 and C2 are different circles d. none of these

65. Angles in the same segment of a circle are

a. complementary b. such that one is greater than the other
c. equal d. none of these

66. The angle in the major segment of a circle is

a. obtuse b. acute c. right d. none of these

67. Two equal circles intersect in A and B. Through B a straight line perpendicular to AB is drawn to meet the circumferences in X and Y. Then,

a. $AX > AY$ b. $AX < AY$ c. $AX = AY$ d. none of these

68. A circle cannot cut a straight line in

a. more than one point b. more than two points
c. two points d. none of these

69. P is the centre of a circle of radius r and distance between the centre of the circle and any point R on a given line PR. The line does not intersect the circle, when

a. $PR = r$ b. $PR < r$ c. $PR > r$ d. none of these

70. A circle is a locus of

a. a point b. arcs c. sectors d. none of these

71. The lengths of the two tangents drawn from an external point to a circle are

a. unequal b. equal c. in the ratio 1 : 3 d. none of these

72. Chord PQ of a circle is produced to O. T is a point such that OT becomes a tangent to the circle. Then,

a. $OQ^2 = OT.OP$ b. $OP^2 = OT.OQ$
c. $OT^2 = OP.OQ$ d. none of these

73. If three circles with equal radii touch each other externally, then the triangle formed by joining their centres is

a. isosceles b. equilateral
c. obtuse angled d. none of these

74. P is the midpoint of an arc APB of a circle. The tangent at P is

a. perpendicular to the chord AB b. intersects the chord AB
c. parallel to the chord AB d. none of these

75. From the vertices B and C of a triangle ABC, perpendiculars are drawn on the opposite sides meeting them at D and E respectively. If these perpendiculars intersect each other at O, then,
a. BO.OD = EO.OC
b. EO.OD = BO.OC
c. OC.OD = BO.OE
d. none of these

76. Two circles cut at A and B and a straight line PAQ cuts the circles at P and Q. If the tangent at P and Q intersect each other at T, then,
a. P, A, Q, T are concyclic
b. P, B, Q, T are concyclic
c. both a & b
d. none of these

77. A line joining any two points on the circumference of a circle is called a/an
a. arc b. chord c. sector d. segment

78. The figure formed by a chord and an arc in the circle is called a
a. segment b. chord c. sector d. none of these

79. An angle with vertex on the circle formed by secant ray and a tangent ray has measure equal to
a. half the measure of the angle subtended by the intercepted arc at the centre
b. the measure of the intercepted arc
c. 1/4 the measure of the intercepted arc
d. none of these

80. An isosceles triangle ABC is inscribed within a circle; AB = AC and AP is a tangent at A to the circle; then
a. AP is perpendicular to BC
b. AP is parallel to BC
c. AP intersects BC at some point
d. none of these

Answers

1. c	**2.** b	**3.** d	**4.** a	**5.** b	**6.** a	**7.** b	**8.** c
9. a	**10.** a	**11.** c	**12.** b	**13.** b	**14.** b	**15.** a	**16.** b
17. b	**18.** a	**19.** d	**20.** d	**21.** b	**22.** a	**23.** a	**24.** b
25. b	**26.** a	**27.** d	**28.** a	**29.** b	**30.** c	**31.** b	**32.** b
33. b	**34.** a	**35.** c	**36.** c	**37.** c	**38.** b	**39.** a	**40.** b
41. c	**42.** b	**43.** b	**44.** d	**45.** c	**46.** c	**47.** d	**48.** c
49. b	**50.** b	**51.** c	**52.** a	**53.** c	**54.** b	**55.** c	**56.** c
57. a	**58.** c	**59.** c	**60.** c	**61.** c	**62.** b	**63.** c	**64.** b
65. c	**66.** b	**67.** c	**68.** b	**69.** c	**70.** a	**71.** b	**72.** c
73. b	**74.** c	**75.** a	**76.** b	**77.** b	**78.** a	**79.** a	**80.** b

Solutions
Concepts 2

1. The area equals one half the base times the height.
$A = {}^1/_2$ (base)(height); $30 = {}^1/_2$ (10)(DB); 60 = 10DB; 6 = DB
Right angled triangle has a 45^o angle so it is isosceles, with DB equal to AB. Thus we know that AB is also 6 units long. Segments BC equals AC minus AB. Using the theorem of Pytahgoras for triangle DBC, $(DB)^2 + (BC)^2 = (DC)^2 \therefore DC = 2\sqrt{13}$

2. The sum of all angles around point C must be equal to 360^o. The three angles in between the isosceles triangle add up to 90 + 100 + 110 = 300^o. Therefore the three vertex angles of the triangles must add up to 60^o. The triangles are congruent, the vertex angles must be equal and each one must be $20^{o.}$. In triangle ABC,
CAB + CBA + 20 = 180 But CAB = CBA
$\therefore$2CAB + 20 = 180 $\therefore$ CAB = 80

3. First find the length of BD.
Area = 0.5(base)(height) $\therefore$ 5 = 0.5($\sqrt{2}$)(BD) $\therefore$ BD = $5\sqrt{2}$
Since the two triangles are congruent, BD equals BG. GD is twice the length of BD = $10\sqrt{2}$

4. In Δs GAB and GCD, GB/GD = AB/CD $\Rightarrow$ 2/GD = 2/3 or GD = 3.
In Δs GCD and GEF, GD/GF = CD/EF $\Rightarrow$ 3/GF = 3/5 or GF = 5. DF = GF – GD = 2.

5. In triangle KLM, the sum of angle LKM and angle KLM is $125^{o.}$ $\therefore$ angle KML is 180 - 125 = 55^o. In triangle LMN, the sum of angle MLN and angle LMN is 105^o. $\therefore$Angle LMN = 75^o. This is equal to opposite vertical angle JNK. The sum of angles x and y equals (180 - 75) = 105^o.

6. Both triangles ABC and CDE are isosceles. Base angles BAC and BCA are equal. They are both equal to 74^o. Angle DEC and angle DCE equal 37^o . Angle BCD is equal to (180 - (74 + 37) = 69^o

7. The four congruent equilateral triangles AFB, FBD, CBD and EFD are each equal to ${}^1/_4$ area of triangle AEC. Trapezium AFDC is made up of three of them and its area is equal to ${}^3/_4$ of the area of AEC = 24.

8. The diagonals divide the rectangle into four different half rectangles and four different quarter rectangles. Four smaller triangles are formed by the two short lines, making twelve triangles in all.

9. Triangle XYA is a 30-60-90 triangle. Hypotenuse is 4. $\therefore$ XA = 2 Base = 5. Height = 4y
$\therefore$ Area = 10y

10. Vertical angles are equal. Angle AZB is equal to angle YZX = 45^o. The sum of three angles of a triangle = 180^o. $\therefore a + b + 45^o = 180^o$ $\therefore a + b = 135^o$.

11. Triangle ABD is a 30-60-90 triangle, altitude BD = AB$\sqrt{3}$/2 . $\therefore$ area = $20\sqrt{3}$

12. Angle x = angle AFD (alternate angles)
angle AFC is complementary to angle y. $\therefore x + y = 90^o$

13. In an equilateral triangle altitude and the angle bisector are the same, AE is also an altitude. BD is also an angle bisector. All the altitudes or angle bisectors in an equilateral triangle are congruent AE = BD. $\therefore$ all 3 statements are true.

14. The area of the region that the two squares overlap is ${}^1/_4$ the area of each square. AB = 8, the area of the figure is $8^2 + 8^2 - ({}^1/4)\, 8^2 = 112$.

15. The side of square ADGH is 2. The area of BCEF is equal to sum of the shaded and unshaded regions = 4 + 21 = 25. A side of this square is 5. ABCD is a trapezoid since AD is parallel to BC. The area is equal to 0.5(a + b)h. a = 2 , b = 5The altitude is the line segment YZ, which is perpendicular to both the bases of the trapezoid. WX + XY + YZ = 5.
WX = YZ, XY = 2, 2YZ + 2 = 5, YZ = 1.5. The area of ABCD is = 0.5(2 + 5)1.5 = 21/4

16. Since the area of a triangle is 0.5xbasex height, the area of the two triangles are ba and 1.5ba respectively., which added together give an area = 2.5ab.

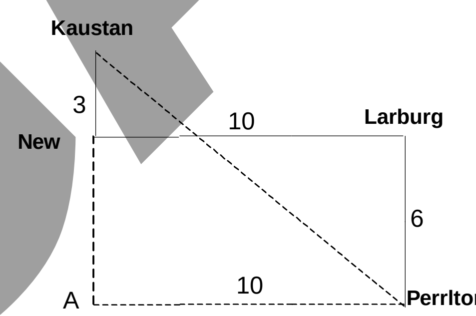
17. Triangle ABD is a 30-60-90 triangle, AB is equal to $6\sqrt{3}$. E is the bisector of BD, EF will be half of AD and EG will be half of AB. The dimensions of the small rectangle are $\therefore$ $3/3\sqrt{3}$ and its area is $9\sqrt{3}$.

18. The area of triangle ABE = 0.5(AF)(BE) = 0.5 x 15 x 18 = 135 = 0.5(AD)(AB)
$\therefore$ (AD)(AB) = 270 = area of rectangle ABCD.

19. The area of the lower rectangle is 2x3 -= 6. The area of the upper left rectangle is
1x3 = 3. The area of the upper right rectangle is 1x1 = 1. The sum of the areas is 10.

20. x = DF and y = FE. Area of triangle DFE = 0.5xy. DG = DF/3 = x/3 = AC; BC = 2FE/3 = 2y/3 $\therefore$ the area of the right triangle ABC is 0.5(x/3)(2y/3) = xy/9;
The ratio of the area of DFE to ABC is (xy/2) : (xy/9) = 2 :9

21. The distance from Perriton to Kauston is the straight line joining those towns. This is a hypotenuse of the right triangle bordered by Perriton, Kauston and Point A. Using the Pythagoras Theorem, $(3 + 6)^2 + 10^2$ = hypotenuse^2 $\therefore$ h = $\sqrt{181}$



22. The length of the rectangle Y is c, and the width is equal to a - d. The area is c(a - d) = ac - cd.

23. The area of the square is s^2. The area of the equilateral triangle of side s is $\sqrt{3}s^2/4$. The area of the rectangle is $2s^2$. The fraction of the rectangle that remains uncovered is 1 - (fraction that is covered). Therefore, the uncovered area = $1 - (s^2 + \sqrt{3}s^2/4)/8 = (4 - \sqrt{3})/8$.

24. a = 8, x = 10 let the other diagonal be y. $a^2 = (x/2)^2 + (y/2)^2$ $\therefore$ y = $2\sqrt{39}$. Area of rhombus = (xy)/2 = 10 x 2 $\sqrt{39}$/2 = 20 $\sqrt{39}$ sq. metres. Area of both the surfaces = 20 $\sqrt{39}$ sq. metres. Cost of painting both the surfaces = Rs.100 $\sqrt{39}$.

25. PR^2 = PQ. PS $\quad\therefore q^2 = 25 \times 16 \quad \therefore q = 20$
QR^2 = QP.QS $\quad\therefore p^2 = 25 \times 9 \quad \therefore p = 15$
SR^2 = SP.SQ $\quad\therefore h^2 = 16 \times 9 \quad \therefore h = 12$

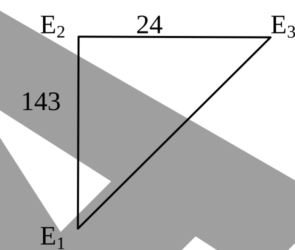
26. A(ADB) = 32/2 16 sq. cm (diagonal bisects the area)
A(ADM) = 16/2 = 8 (DM is the median of ADB)
A(AMN) = 8/2 = 4 (MN is the median of AMD)

27. Area of the incircle = $\pi r^2 = 154$. So In-radius = 7 cm. But In-radius = $\Delta/S = (\sqrt{3}X^2/4) / (3X/2) = X / (2\sqrt{3}) = 7$. Therefore X i.e. the side of the equilateral triangle is equal to $14\sqrt{3}$ cm. Therefore the Circumradius = $X^3 / (4x\sqrt{3}X^2/4) = 14$. Therefore the ratio of the radius of the circumcircle to that of the incircle is 2:1.
Inradius = Δ/s & Circumradius = $abc/4\Delta$
Ratio of Inradius/ Circumradius = $\sqrt{3}a^2.2.4.\sqrt{3}.a^2/(4.3a.a^3.4) = 1/2$

28. Length of the diagonal of □ ABCD = $\sqrt{2} \times 10/\sqrt{2} = 10$ cm. In ΔABD, M_4 and M_1 are mid points of AD and AB. So $l(M_4M_1) = 10/2 = 5$. But □$M_1M_2M_3M_4$ is a square. Therefore the required area of □$M_1M_2M_3M_4$ is equal to 25 cm^2.

29. Length of the ladder is equal to 50’ and now its lower end is now 40’ away from the bottom of the wall. So the height at which the upper end of the ladder will be 30’ by Pythagorus theorem.

30. Fish swims 143 m due north from E_1 to reach E_2 on the edge of the pond then turns in 90° i.e. east and swims 24 m to reach E_3 on the edge which must be diagonally opposite to the starting point. So the diameter of the pond is equal to $E_1E_3 = \sqrt{[(143)^2+(24)^2]} = 145$ m.



31. Let $m\angle PQS = m\angle SQR = \alpha$ and $m\angle PRS = m\angle SRQ = \beta$.
Therefore inΔPQR, we have $66°+2\alpha+2\beta = 180°$. So $\alpha+\beta = 57°$. But in ΔQSR, $m\angle QSR+\alpha+\beta = 180°$. SO $m\angle QSR = 180-57 = 123°$.

32. By Appollonius theorem, $6^2+8^2 = 2(5)^2+2(QM)^2$. Therefore QM = 5,but M is the midpoint of QR.So l(QR) = 10 units.

33. Suppose, we draw the diagonal PR. We know that the diagonals of a parallelogram bisect each other. So, SQ passes through the mid-point of PR. PU and a part of SQ are two medians of the ΔPSR. The centroid of a triangle divides the median in the ratio 2:1. So, SV = VW = WQ = 5 units.

34. In ΔPRS, l(PR) = l(RS). Therefore $m\angle RPS = 25°$. But $\angle PRQ$ is the exterior angle of the ΔPRS, so $m\angle PRQ = 50°$. In ΔPQR, l(PQ) = l(QR). So $m\angle QPR = 50°$. Therefore in ΔPQR, $m\angle PQR = 80°$.

35. PS is the diagonal of the parallelogram PTSQ. Therefore $A(\Delta PTS) = A(\Delta PSQ)$ and SQ is a diagonal of the parallelogram PSQR, $A(\Delta PSQ) = A(\Delta SQR)$. But A(Trapezium TPQR) = $A(\Delta PTS)+A(\Delta PSQ)+A(\Delta SQR) = 3x13 = 39$ sq. units.

Concepts 3

1. angle DOC = 45^o , area of each sector is 45/360 or 1/8 of the area of the circle with the same radius. Area of sector COD = $\pi 4^2/8 = 2\pi$
Area of sector AOB = $\pi 5^2/8 = 25\pi/8$
Area of the shaded region = $25\pi/8 - 2\pi = 9\pi/8$

2. Area of the square is 1, diameter of the circle is also 1 or the radius is 0.5. Area of the circle is $\pi(1/2)^2 = \pi/4$. The area of the shaded region is $1 - \pi/4$.

3. The figure actually consists of 2 circles and a rectangle. One circle has a diameter of 6 and the other has a diameter of 8. Total area = $\pi(3)^2 + \pi(4)^2 + 6(8) = 25\pi + 48$

4. The side of the square is 2r. The area of the square is $4r^2$. The area of the circle is πr^2. The area of the shaded portion is the difference between the two areas. $4r^2 - \pi r^2 = r^2(4 - \pi)$.

5. The entire circle has 360^o. x represents the number of degrees in angle DBE plus angle ABC. $\therefore 63/360 = 14/x$ $\therefore$ x = 80 as angle DBE = angle ABC $\therefore$ angle ABC = 80/2 = 40^o.

6. The area of the ring is equal to the difference between the areas of the outer circle and the inner circle. Let R be the radius of the outer circle. The area of the large circle is πR^2. The area of the small circle is 16π. $\pi(R^2 - 16) = 176$ $\therefore R = 6\sqrt{2}$ $\therefore$ thickness = $6\sqrt{2} - 4$

7. The distance between any two of the adjacent circles is equal and $\therefore$ AB = BC.

8. The length of a side of the large square is equal to the diameter of the circle. The diameter of the circle is also equal to the diagonal of the small square. Each side of the small square is 2 so the diagonal is $2\sqrt{2}$ = length of the side of the large square.

9. Angle ABC is inscribed angle, it equals half the measure of the intercepted arc(AC). Arc (AC) = 58^o = Angle(AOC). Angle OAD = 90^o. Angle ADO = 180 - (90 + 58) = 32^o. Angle EDF and ADO are vertical angles. $\therefore$ angle EDF = angle EDO = 32^o.

10. PT and RS are both diameters of their respective circles. The arc between two different ends of a diameter always equals 180^o. Arc PT = 180^o. Arc RS = 180^o. $180^o - 180^o = 0$

11. The area of the shaded portion equals the area of the semi circle minus the area of the triangle ABC. Triangle ABC is an isosceles right triangle. $\therefore$ CB = AC = 4. $\therefore$ Area of the triangle = (0.5) x (4)(4) = 8. Using the theorem of Pythagoras, $(AC)^2 + (CB)^2 = (AB)^2$ $\therefore$ AB = $\sqrt{32}$; $\therefore$ radius of circle = $\sqrt{32}/2$ area of the semi circle = $8\pi/2 = 4\pi$. $\therefore$ the area of the shaded region $=4\pi - 8$.

12. Diagonal of the square = $14\sqrt{2}$ cm. $\therefore$ the side = 14 cm. Diameter of the inscribed circle = 14 cm. $\therefore$ the radius = 7cm. $\therefore$ the area $=\pi\ 7^2 = 49\ \pi$ sq. cm.

13. l(direct common tangent) = $\sqrt{[\ 17^2 - (12 - 4)^2\ ]} = \sqrt{(9 \times 25)}$ = 15 cm. l(transverse common tangent) = $\sqrt{17^2 - (12 + 4)^2} = \sqrt{(1 \times 33)} = \sqrt{33}$ cm.

14. PA.PB = PC.PD $\therefore$ 2 x PB = (9 - 3) x 3 = 18; $\therefore$ PB = 9 cm.

15. 42 x 42 - π x 35 x 35 /4 = 801.5 m^2

16. A(ΔOAB) = 12 =2xA(ΔOCA). Therefore A(ΔOCA) = 6. But AC = ½ x AB = 4. But A(ΔOCA) = ½ x OC x AC = ½ xOCx4 = 6. Therefore OC = 3. Therefore l(OA) = 5, by Pythagoras theorem in ΔOCA.

17. Suppose the radius of the circle is ‘r’. Then we have by tangent-secant theorem $(12)^2 = (9)(9+2r)$. On solving, r = 3.5units.

18. We have [l(OQ)+r] [l(OQ)-r] = [l(PQ)] [l(RQ)]. Therefore [l(OQ)+9] [l(OQ-9] = [6+4] [4]. On solving we get l(OQ) = 11.

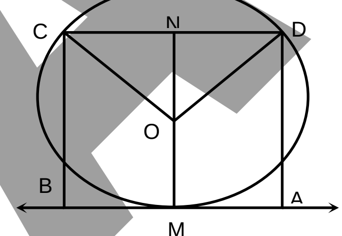
19. □ABCD is a cyclic quadrilateral so $m\angle DAB = 180-120 = 60°$. $m\angle ADB = 90°$. Therefore in ΔADB $m\angle ABD = 180-90-60 = 30°$.

20. The diameter of the circle is also the diagonal of the rectangle and is 13cm long. If one side of the rectangle is 12 cm long, then by Pythagoras theorem, the other side must be 5 cm. Therefore, the area of the rectangle is 60 cm^2.

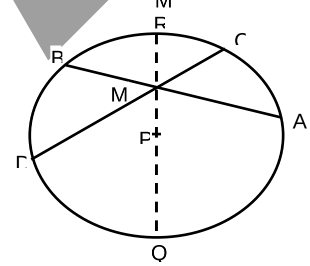
21. Suppose radius of the inscribed circle is r. If we draw perpendiculars on the sides of the quadrant from the centre of the inscribed circle we get a square of side equal to r. Therefore the diagonal of the square is equal to $r\sqrt{2}$. But $r\sqrt{2}+r = R$. Therefore $r = R/(1+\sqrt{2}) = R(\sqrt{2}-1)$.

22. Let PQ intersect AB in the point R. $\Delta APR \sim \Delta BQR$. So, AP/AR = BQ/BR. Therefore, 3/AR = 5/BR, i.e. AR/BR = 3:5.

23. Let AB = x. $\Delta OCN \cong \Delta ODN$. Therefore, CN = DN = x/2. Therefore, $\sqrt{\{25-(x/2)^2\}} + 5 = x = MN$. On solving the above equation, we get x = 8. Therefore, the area of the square is 64.



24. Let RQ be the diameter of the circle passing through M. RM = 8. Therefore, CM x DM = RM x MQ = 8 x 18 = 144 cm^2.



25. By Pythagoras theorem, $AC^2 = 36 + 64$, i.e. AC = 10. Inradius = Δ/s = (1/2 x 6 x 8)/{1/2(6 + 8 + 10)} = 2. Therefore, the area of the in – circle = 4π.

Concepts 4

1. Using two intercept form i.e. $x/a + y/b = 1$, Required equation of the line is $x/10 + y/10 = 1$ i.e. $x + y = 10$.
2. The line passes through the points (- 3, 7) and (0, -2). ∴ Slope of the line = $(-2-7)/(0+3) = -3$. ∴ Using slope intercept form, equation of the line is : $y = -3x - 2$ i.e. $3x + y + 2 = 0$.

3. Slope of PQ : $[1 - (-3)] / [6 - 0] = 2/3$ Slope of RS : $[2 - (-4)] / [5 - (-4)] = 2/3$ Since the slopes of the two lines are equal, the lines are parallel.

4. The equations of the two lines can be rewritten as : $y = 3x + 4$ and $y = (-2)x + 3$. The slopes of the two lines are thus 3 and (-2). If θ is the required acute angle, we have, $\tan\theta = \left| [3 - (-2)] / [1 + 3(-2)] \right| = |-1|$. The required angle is $\tan^{-1}|-1| = 45^0$.

5. Since the line is parallel to y-axis, all points on the line have the same x-coordinate. Since the given point has 4 as the x-coordinate, all other points also have the same x- coordinate. Hence the equation of the line becomes $x = 4$. Using the above reasoning, the line perpendicular to the y-axis has the same y-coordinate for all its points and the equation is $y = -6$.

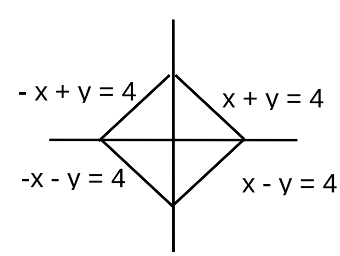
6. We have $(PQ)^2 = [(-2) - 4]^2 + [2 - 5]^2 = 45$ and $(PR)^2 = [(-2) - 3]^2 + [2 - (2 + 2\sqrt{5})]^2 = 45$. Since two of the lines have equal lengths, the triangle is an isosceles triangle.

7. Let the points be A(0, 0), B(4, 3), C(3, 5) and D(-1, 2). Assuming A and B to be adjacent vertices, we have, $AB = \sqrt{(0 - 4)^2 + (0 - 3)^2} = 5$, and $CD = \sqrt{(3 - (-1))^2 + (5 - 2)^2} = 5$. Also, $BC = \sqrt{(4 - 3)^2 + (3 - 5)^2} = \sqrt{5}$ and $DA = \sqrt{(-1 - 0)^2 + (2 - 0)^2} = \sqrt{5}$. Thus opposite sides are equal, i.e. the quadrilateral is at least a parallelogram. Also, $AC = \sqrt{(0 - 3)^2 + (0 - 5)^2} = 5$ and $(AC)^2 \neq (AB)^2 + (BC)^2$. Hence the quadrilateral is not a rectangle. ∴ The given quadrilateral is a parallelogram.

8. A and B are points on the X-axis. Since the ΔABC is an equilateral triangle, the third vertex C will lie on the perpendicular bisector of AB. The mid-point of AB is $((a + 3a) / 2, (0 + 0) / 2)$. Let this point be D. ∴ D(2a, 0). (AB) = 2a = side of the equilateral triangle. ∴ The altitude is $(\sqrt{3}/2)(AB) = a\sqrt{3}$, i.e. $(CD) = a\sqrt{3}$. Since $(CD) \perp$ X-axis, the x-coordinate of C and D is the same, and since D is on the X-axis, the y-coordinate of C is equal to length of CD. ∴ The coordinates of C are $(2a, a\sqrt{3})$ or $(2a, -a\sqrt{3})$.

9. The perpendicular distance of the vertex from the line is the altitude of the triangle. ∴ Altitude $= \left| (3(3) + 7(-5) - 12) / \sqrt{[(3)^2 + (7)^2]} \right| = 38/\sqrt{58}$. Since the base is given to be 12 units, area $= 1/2\, [(12) \times (38/\sqrt{58})] = 228/\sqrt{58}$ sq. units.

10. The four possible lines of the above locus are $x + y = 4$; $x - y = 4$; $-x - y = 4$ and $-x + y = 4$. The four lines can be represented on the coordinate axes as shown in the figure. Thus a square is formed with the vertices as shown. The side of the square is $\sqrt{[(0-4)^2 + (-4-0)^2]} = 4\sqrt{2}$. The area of the square is $(4\sqrt{2})^2 = 32$ sq. units.



11. $l(AB) = \sqrt{[(6-0)^2 + (0+8)^2]} = 10$. Therefore area of the □ABCD $= 10^2 = 100$ sq. units.

12. □PQRS is a square, so the diagonal QS makes an angle of 45° with SR. But SR is parallel to the X-axis. Therefore the slope of the diagonal SQ = tan 45° = 1.

13. Since the points A,B and C are collinear, Slope of AB $= (5-2)/(2+7) = 1/3$ = Slope of BC $= (3-2) / (a+7)$. On solving we get $a = -4$.

14. Suppose the line makes the equal intercepts of a each on X-axis and Y-axis. Therefore the equation of the line is x/a+y/a = 1. Since the line passes through (-2,4), we have -2/a+4/a = 1. On solving we get a = 2. Therefore the equation of the line is x/2+y/2 = 1 i.e. x+y = 2.

15. $$A(\Delta ABC) = \frac{1}{2}\begin{vmatrix} 4 & 8 & 1 \\ 20 & 6 & 1 \\ -2 & 10 & 1 \end{vmatrix} = 10.$$

If we join mid points of AB, BC and CA, we get four equal triangles. Therefore area of the triangle whose vertices are mid points of AB, BC and CA is 10/4 = 2.5 sq. units.

16. Height of the trapezium = Distance between the two given lines = $(20-15)/\sqrt{[(6-3)^2+(8-4)^2]}$ = 5/5 = 1. Therefore the area of the trapezium = ½ (6+2) x 1 = 4 cm^2.

17. The centroid of the ΔPQR has coordinates (1/2,1/3). Therefore 1/2 = (x+0+1/2)/3 and 1/3 = (1+y+0)/3. On solving we get x = 1 and y = 0. Therefore slope of PQ = (0-1)/(0-1) = 1 and slope of QR = (0-0)/(1/2 -0) = 0. Suppose θ is the angle between PQ and QR, then tanθ = (1-0)/(1+1x0) = 1. Therefore θ = m∠PQR = 45°.

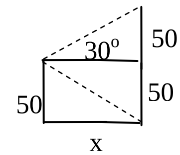
18. Equation of the line can be written as x/2+√3y/2 = 12 i.e. xCos60°+ySin60° = 12√5. Comparing this with the Normal form xCosα+ySinα = p, we get the inclination is equal to 60°and the perpendicular distance from the origin is equal to 12√5units.

19. Segment AB is divided into five equal parts at P, Q, R and S. Therefore S divides PR externally in the ratio 3:1. By Section formula co-ordinates of S are [(4x3-8x1)/(3-1), (16x3-12x1)/(3-1)] i.e. (2,18). Now inclination of the line is equal to 135°. Therefore slope of the line is equal to tan135°, i.e. -1. By Slope-Point form we have y-18 = -1(x-2) i.e. x+y =20.

20. Slope of the line x + 3y = -7 is –1/3. On solving the equations for the three lines, we get the vertices of the triangles as (-1, 2), (3, 5), (1, 8). So, the centroid of the triangle is (1,5). Using the slope – point form of the equation of a line, the required equation is y – 5 = (-1/3)(x – 1), i.e. x + 3y = 16.

Concepts 5

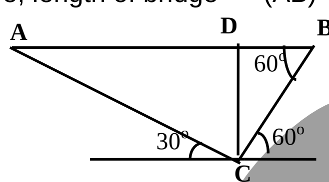
1. From the figure it is obvious that the height of the tower is twice that of the building. Hence the height of the tower = 100 m. Also, if x is the distance between the two, $\tan 30^0 = 50 / x = 1 / \sqrt{3}$ ∴The distance = x = $50\sqrt{3}$ m.



2. Since $\angle ONP = 45^0$, and the pole OP is vertical, i.e. perpendicular to the ground, $\angle NOP = 90^0$ and hence $\angle NPO = 45^0$. The ΔPON is an isosceles triangle, with (PO) = (ON).
Thus, (ON) = 10 m. Also $\tan \angle OMP = \tan 30^0 = (OP/ OM) = 1/\sqrt{3} = 10 / OM$
(OM) = $10\sqrt{3}$. (MN) = (OM) + (ON) = $10\sqrt{3} + 10$.

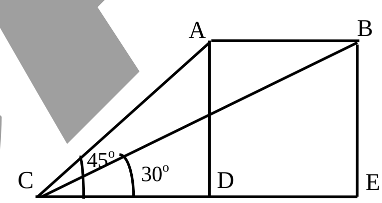
3. Let w be the width of the river and h : the height of the tree.
∴ $\tan 45^0 = h / w$ and $\tan 30^0 = h / w + 50$
∴ From the above, $w / (w + 50) = \tan 30^0 / \tan 45^0 = 1 / \sqrt{3}$ ∴ $w = 50 / [\sqrt{3} - 1]$

4. We have, length of bridge = (AB) = $500\sqrt{3}$. Also, $\angle ACB = 180^0 - (\angle ACD + \angle DCB) = 90^0$



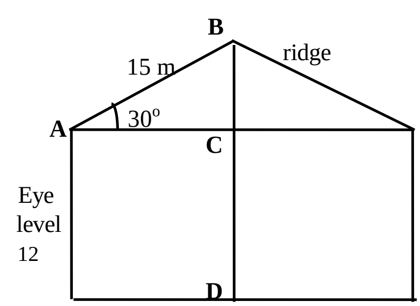
∴ $\cos 60^0$ = (CB /AB) ∴ (CB) = $250\sqrt{3}$.
∴ (CD) = (CB) $\cos 30^0$ = $250\sqrt{3} \times \sqrt{3}/2$.
∴ Height of bridge = (CD) = 375 m.

5. Let A and B be the successive positions of the airplane and let (DE) be the distance covered in 10 seconds. We have, CD = AD / tan 45 and CE = BE / tan 30. CD = 100 m, and CE = $100\sqrt{3}$ m. As AD = BE = 100 m i.e. the altitude of the airplane, DE = CE - CD = $100(\sqrt{3} - 1)$ m = 73.2 m. The speed = 73.2 m/ 10 s = 7.32 m/s.

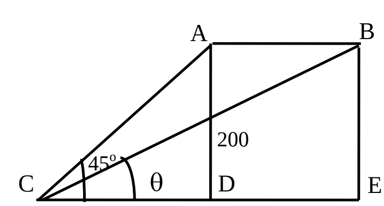


6. If h is the height of the post, we have, $h^2 = (100)^2 - (50)^2 = 50 \times 150$. The height of the post is $50\sqrt{3}$ m.

7. We have (AB) = 15 m and (BC) = 15 sin 30^0. ∴ BC = 7.5 m. The height of the ridge above the ground level is BC + CD = 7.5 + 12 = 19.5 m.



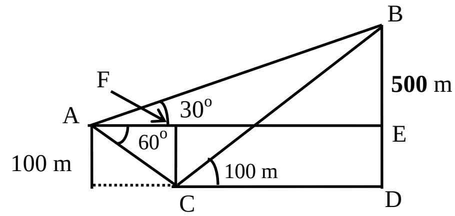
8. Let C be the position of the submarine, and A be the initial position of the ship and B its position when the missile hits. Thus the missile travels from C to B.
We have, obviously, (BE) = (AD) = 200 m. Let θ^0 be the angle at which the missile is fired.
∴(CB) = (BE) / sin θ = 200 / sin θ. (CD) = (AD) = 200 m, since Δ ACD is an isosceles right triangle.



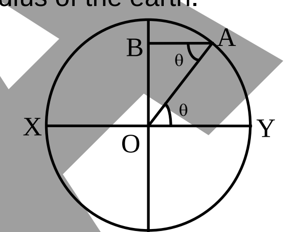
Also, (AB) = (DE) = (CE) - (CD) = [(BE) / tanθ] - (CD) = [200 / tan θ] - 200 = 200 [(1/ tan θ) - 1]. The ship and the missile cover distances (AB) and (CB) respectively in the same time.
∴ $200 (1/ \tan \theta - 1) / [20\sqrt{3} - 1] = (200 / \sin \theta) / 40$. Simplifying the equation we get, $\cos \theta - \sin \theta = (\sqrt{3} - 1) / 2$ ....(a) Squaring both sides, we get, $\cos^2 \theta + \sin^2 \theta - 2 \cos \theta \sin \theta = 1 - \sqrt{3} / 2$. ∴$2 \cos \theta \sin \theta = \sqrt{3}/2$ ∴ $1 + 2 \cos \theta \sin \theta = 1 + \sqrt{3} / 2$. ∴ $\cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta$

$= 1 + \sqrt{3} / 2$. $\therefore (\cos\theta + \sin\theta)^2 = ((1 + \sqrt{3}) / 2)^2$ $\therefore \cos\theta + \sin\theta = (\sqrt{3} + 1) / 2$ ....(b). Adding (a) and (b), we get, $2\cos\theta = \sqrt{3}$ $\therefore \cos\theta = \sqrt{3} / 2$ $= 30^0$.
Hence the missile should be fired at angle of $30^{0.}$

9. Let A, B, C be the positions of the observer, the aircraft,and the boat respectively. (BE) = (BD) - (ED) where (ED) = height of the cliff = 500 - 100 = 400 m. AE = BE / tan 30 = $400\sqrt{3}$ m. AF = FC / tan 60 = $(100 / \sqrt{3})$ m. Also (CD) = (FE) = (AE) - (AF) = $400\sqrt{3} - (100 / \sqrt{3}) = 1100 / \sqrt{3}$ m. If θ^0 is the angle of elevation of the aircraft as seen from the boat, $\tan\theta = (BD / CD) = 500 / (1100 / \sqrt{3}) = 5\sqrt{3} / 11$ $\therefore \theta = \tan^{-1}(5\sqrt{3} / 11)$.



10. If r is the radius of the earth, the equator is $2\pi r$. For a circle we have : $s = r\,\theta$, where θ^c = angle subtended at the centre by arclength 's'. In this case, s = arc YA = $(2\pi r / 6) = \pi r / 3$ $\therefore \pi r / 3 = r\,\theta$. $\therefore \theta = (\pi / 3)^c = 60^0$. If r' = (BA) is the radius of the latitude at A, we have, $\angle$ OAB = θ = 60^0, since the latitude is parallel to the equator. Also (OA) = r = radius of the earth.
$\therefore$ r' = r cos θ = r cos 60 = r / 2. Since the radius of the latitude is half the radius of the equator its length is also half the equator. $\therefore$ The required ratio is 1 : 2.



11. $\sin^6 A + \cos^6 A + 3\sin^2 A\cos^2 A = (\sin^2 A + \cos^2 A)^3 - 3\sin^4 A.\cos^2 A - 3\sin^2 A\cos^4 A + 3\sin^2 A\cos^2 A = 1 - 3\sin^2 A.\cos^2 A (\sin^2 A + \cos^2 A - 1) = 1$.

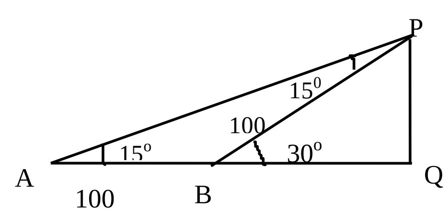
12. (cot A + tan B) / (tan A + cot B), convert tan A and cot A in the form of sin A and cos A and simplify to get the desired result.

13. (sin A + cos A).(tan A + cot A)
= (sin A + cos A).[(sin A / cos A) + (cos A / sin A)]
Multiply the two and simplify to get the desired result.

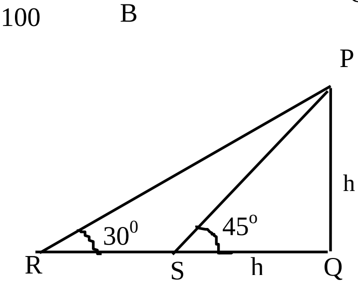
14. $(1 - \tan^2 A) / (1 + \tan^2 A) = \cos 2A$, hence, the given expression reduces to cos 30° which is equal to $\sqrt{3} / 2$.

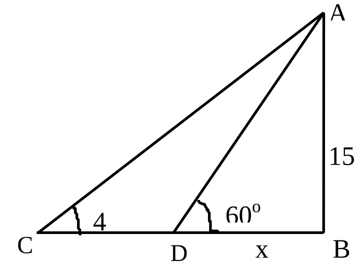
15. tan (x + y) = (tan x + tan y) / (1 - tan x. tan y), put in the given values of tan x and tan y and the expression simplifies to 1, implying that x and y sum up to $\pi / 4$.

16. If the height of the pillar be h, then, from the figure AB = BP = 100, and h / BP = sin 30^0. h = 100 x 1/2 = 50m. Also h / BQ = tan 30^o = $1/\sqrt{3}$, BQ = $h\sqrt{3} = 50\sqrt{3}$ m, hence AQ = AB + BQ = $100 + 50\sqrt{3} = 50(2 + \sqrt{3})$.

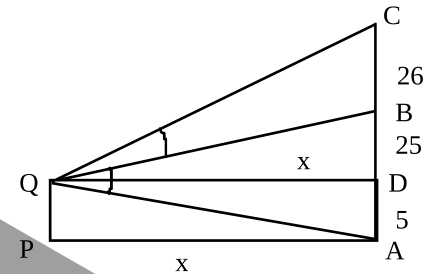


17. According to the question, RS = 60m,
h / (h + 60) = tan 30^o = $1/\sqrt{3}$, solve for h to get,
h = $30(\sqrt{3} + 1)$m.

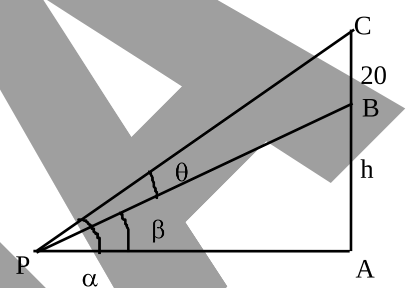




18. If AB be the cliff and the boat travels from D to C, $\tan 60^o = 150 / x$, $x = 50\sqrt{3}$, Now, $\tan 45^o = 150 / (CD + x)$, or $CD + x = 150$, $CD = 150 - 50\sqrt{3}$, Speed of the boat = $(150 - 50\sqrt{3}) / 2 = 25(3 - \sqrt{3})$ m/min.



19. $\angle CQB = \angle AQB$, hence QB is bisector of angle AQC, such that it divides the base AC in the ratio of the arms of the angle $AB / BC = QA / QC$, or $25 / 26 = (QD^2 + DA^2)^{1/2} / (QD^2 + DC^2)^{1/2} = (x^2 + 25)^{1/2} / (x^2 + 46^2)^{1/2}$, solve the equation to get the value of x = 160 m.

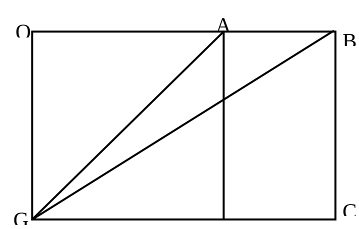


20. $\tan\theta = 1/6$, $\tan\beta = h / 70$ and $\tan\alpha = (h + 20) / 70$
$\alpha = \theta + \beta$, $\tan\alpha = (\tan\theta + \tan\beta) / (1 - \tan\theta.\tan\beta)$
$(h + 20) / 70 = (1/6 + h/70) / (1 - h/420)$,
solve for h to get, h = 50.

21. We have BC = 28 ¼ x4 = 113 and AC = 56x2 = 112. Now by Pythagorus theorem $AB = \sqrt{[(113)^2 - (112)^2]} = 15$.

22. By Pythagoras theorem, we have $BE = \sqrt{[(40)^2 + (9)^2]} = 41$. Now since EF || BC, we have AE/BE = AF/FC. Therefore 82/41 = 90/FC. On solving we get FC = 45.

23. By using Cosine rule in ΔPQR, we have $\cos\angle PQR = [(PQ)^2 + (QR)^2 - (PR)^2] / [2(PQ)(QR)] = (9 + 16 - 13) / (2x3x4) = ½$. Therefore $m\angle PQR = 60°$.



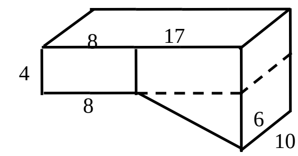
24. We have $m\angle AGC = 45°$ and $m\angle BGC = 30°$ and $BC = \sqrt{3} = AD$. We have in ΔAGD, $AD = GD = \sqrt{3}$. Let S be the speed of the plane in km/hr. Therefore $AB = 10\sqrt{3}(\sqrt{3} - 1) \times 3600S$. Now in ΔBGC, $\tan 30° = BC/GC = \sqrt{3}/[\sqrt{3} + 10\sqrt{3}(\sqrt{3} - 1)S] = 1/\sqrt{3}$. Therefore on solving we get S = 360km/hr.

Concepts 6

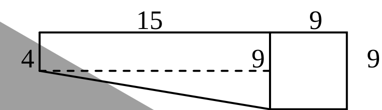
1. Volume of cube = 8 x 8 x 8 cm^3 = 512 cm^3. Original Volume of water = 25 x 20 x 5 = 2500 cm^3. New volume = 2500 + 512 = 3012 cm^3. Actual volume to be reached is 25 x 20 x 8 = 4000 cm^3. Water to be poured = 4000 - 3012 = 988 cm^3.

2. Let the depth of the rainfall be d. 5 x 7 x 10^4 x d = 42 x 20 x 50. $\therefore$ d = 0.12 cm = 1.2 mm.

3. Volume of water = 25 x 10 x 4 + 0.5 x 6 x 17 x 10 = 1510 ft^3
let height at 16.5 ft = 4+x. Also 17/6 = 8.5/x hence, x = 3 ft.
$\therefore$ total depth = 4+3 = 7 feet.



4. The adjacent figure shows the side view of the swimming pool. V = 15 x 10 x 4 + 5 x 10 x15/2 + π x 5^2 x 9/2 = 975 + 225/2 π ft^3



5. Outside surface area = $2\pi rh$ = 2 π x 35/2 x 70 = 7700 cm^2. Inside surface area = $2\pi r_2 h$ = 2 π x (17.5 - 3.5) x 70 = 6160 cm^2. Upper + Lower Surface Area = $2\pi(r^2 - r_2^2) = 2\pi(17.5^2 - 14^2)$ = 693 cm^2. Total Surface area = 14553 cm^2. Cost = 14553 x 5/100 = Rs. 727.65.

6. Length = 350 cm. External d = 2.4, r_1 = 1.2 cm $\therefore$Internal radius r_2 = 1cm. Volume of lead = $\pi(r_1^2 - r_2^2)h$ = 484 cc. 1 cc weighs 11.4 gms $\therefore$ 484 cc weighs 484 x 11.4 = 5517.6 gm

7. $^4/_3\pi r^3 = 4\pi r^2$. $\therefore$r = 3, $\therefore$ diameter = 6.

8. Volume of metal = $^4/_3\ \pi(r_1^3 - r_2^3) = {}^4/_3\ \pi(9^3 - 8^3)$ = (31 x 4 x 22)/3. Cost of shell = (1.80)x(31 x 4 x 22)/3 = 1636.80.

9. Let the radius of the third ball be r cm. Volume of this ball = $^4/_3\pi(3/2)^3 - \{^4/_3\pi(3/4)^3 + {}^4/_3\pi(1)^3\}$ = 125 π / 48. $\therefore$ $4\pi r^3$ / 3 = 125 π /48 $\Rightarrow$ r = 5/4 $\therefore$ diameter = 5/2.

10. Metal required = $(\pi rl + \pi r^2)$ sq. cm. where r = 7 cm. and l = $\sqrt{(7^2 + 24^2)}$ = 25 cm. Substituting the values we get metal required = 704 cm^2.

11. Since cost of plating per unit area is same, Cost $\propto r^2$. $\therefore(r_1 / r_2)^2$ = 215.6 / x = $(7/8)^2$ $\Rightarrow$ x = Rs. 281.60.

12. Volume of cylinder of length l and radius (1/200) cm. π x $(1/200)^2$ x l = 1 $\therefore$ l = 127.3 metres.

13. Volume of cube = 100 Volume of cone carved = $^1/_3\ \pi\ (3)^2$ x 7 = 66 cm^3. $\therefore$ Volume of wood wasted = 34 $cm^3$$\therefore$ % wasted = 34/100 x 100 = 34%.

14. Volume of lead = 22 x 22 x 22   Volume of each bullet = $^4/_3\ \pi\ 1^3$
Number of bullets = (22 x 22 x 22)/ $^4/_3\ \pi$ = 2541

15. Volume of pipe = $\pi(2.5^2 - 1.5^2)$ x 100 = π x 1 x 4 x 100 = (22/7) 400
1 cc weighs 21 gm $\therefore$(22/7) 400 cc weigh 400 $\times$ 21 $\times$ 22/7 =26400 gm.

16. When each edge of the cube is increases by 50%, the surface area of the solid increases 2.25 times. Therefore, the sculptor will need 12 x 2.25 = 27 litres of paint which is 15 litres more.

17. Area of the slant surface at the bottom = $(48^2 + 20^2)^{1/2}$ x 24. Number of tiles required. = $(48^2 + 20^2)^{1/2}$ x 24 / (8 x 12 x $10^{-4)}$) = 130000.

18. $(100 / 2)^3$ = 125000.

19. $(\pi \times 9 \times 21) / (\pi / 3 \times 81/4) = 28$ cm.

20. $7 \times 7 \times 14 = 686$ cc.

21. This is equal to the length of the body diagonal $(12^2 + 10^2 + 8^{2)1/2} = 17.5$ cm.

22. $[(d^3 - \pi (d/2)^2. d/2 - \pi/3 (d/2)^2. d/2) / d^3] \times 100 = 47.64$ %.

23. $(12 \times 10 \times 8 - 3 \times 10 \times 6 \times 4) / 80 = 3$ m.

24. $({}^4/_3 \pi \times 27000) / \pi 4 = 9$ m.

25. The circumference of the base of the cylinder is 44 cm. So, the radius of the base is 7 cm. The height of the cylinder is 10 cm. Therefore, the volume of the cylinder is $\pi (7 \times 7 \times 10) = 1540$ cm^3.

26. The volumes of the slab of iron and the cone are equal. So, $(11 \times 10 \times 2) = (1/3) \pi \times 21h$, where h is the height of the cone. Therefore, h = 10 inches.

27. The length of the wire is the perimeter of the triangle = 24 cm. This is also the perimeter of the regular hexagon. Therefore, the area of the regular hexagon is $6 \times (\sqrt{3}/4) \times (24/6)^2 = 24\sqrt{3}$ cm^2.

28. Total surface area of the cube = $6x(4\sqrt{11})^2 = 1056$. Total area of the cutout circles = $6\pi r^2 = 6(22/7) (\sqrt{7})^2 = 132$. Therefore the required area = $(1056 - 132) / 132 = 7$.

29. We have the Ramanujan number = $1729 = (1)^3 + (12)^3 = (9)^3 + (10)^3$. Volume of a sphere = $(4 / 3)\pi r^3$. Therefore one can select either the first and fourth spheres or second and third spheres.

30. If the level of water rises by h, then $\pi \times 8^2 \times h$ = volume of the three spherical balls = $(4/3) \pi \times 4^3$. Therefore, h = 4 cm.